

Table of Derivatives

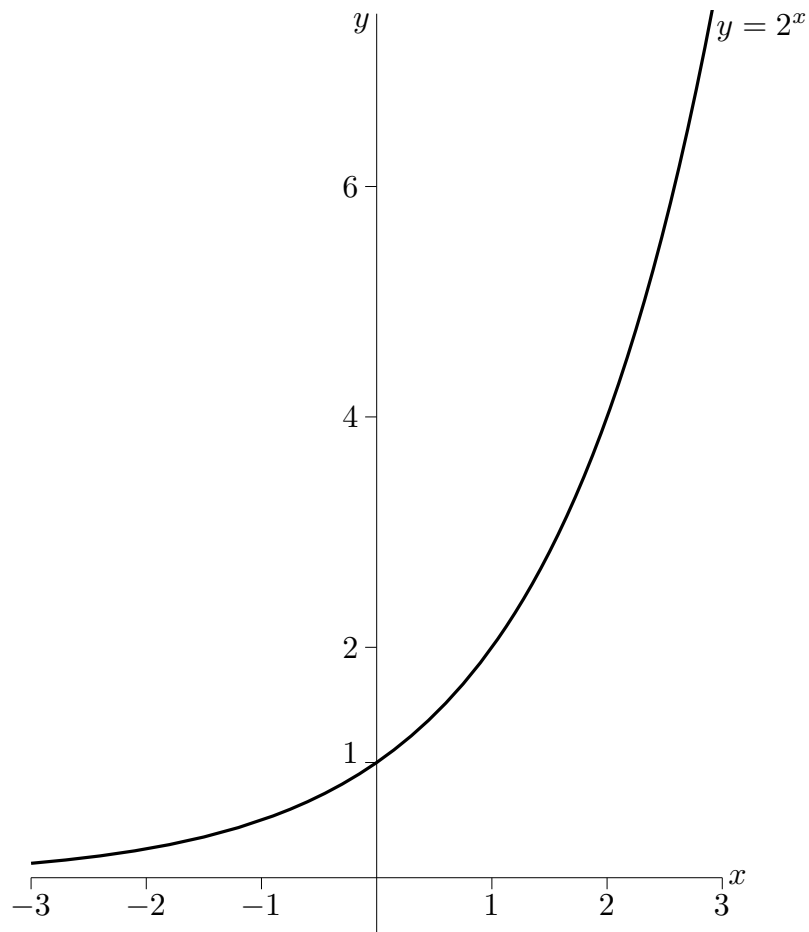
Throughout this table, a and b are constants, independent of x .

$F(x)$	$F'(x) = \frac{dF}{dx}$
$af(x) + bg(x)$	$af'(x) + bg'(x)$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x) - g(x)$	$f'(x) - g'(x)$
$af(x)$	$af'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f(x)g(x)h(x)$	$f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
$\frac{1}{g(x)}$	$-\frac{g'(x)}{g(x)^2}$
$f(g(x))$	$f'(g(x))g'(x)$
a	0
x	1
x^a	ax^{a-1}
$g(x)^a$	$ag(x)^{a-1}g'(x)$
$\sin x$	$\cos x$
$\sin g(x)$	$g'(x) \cos g(x)$
$\cos x$	$-\sin x$
$\cos g(x)$	$-g'(x) \sin g(x)$
$\tan x$	$\sec^2 x$
$\csc x$	$-\csc x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
e^x	e^x
$e^{g(x)}$	$g'(x)e^{g(x)}$
a^x	$(\ln a) a^x$
$\ln x$	$\frac{1}{x}$
$\ln g(x)$	$\frac{g'(x)}{g(x)}$
$\log_a x$	$\frac{1}{x \ln a}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arcsin g(x)$	$\frac{g'(x)}{\sqrt{1-g(x)^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\arctan g(x)$	$\frac{g'(x)}{1+g(x)^2}$
$\operatorname{arccsc} x$	$-\frac{1}{x\sqrt{1-x^2}}$
$\operatorname{arcsec} x$	$\frac{1}{x\sqrt{1-x^2}}$
$\operatorname{arccot} x$	$-\frac{1}{1+x^2}$

Properties of Exponentials

In the following, x and y are arbitrary real numbers, a and b are arbitrary constants that are strictly bigger than zero and e is 2.7182818284, to ten decimal places.

- 1) $e^0 = 1, a^0 = 1$
- 2) $e^{x+y} = e^x e^y, a^{x+y} = a^x a^y$
- 3) $e^{-x} = \frac{1}{e^x}, a^{-x} = \frac{1}{a^x}$
- 4) $(e^x)^y = e^{xy}, (a^x)^y = a^{xy}$
- 5) $\frac{d}{dx}e^x = e^x, \frac{d}{dx}e^{g(x)} = g'(x)e^{g(x)}, \frac{d}{dx}a^x = (\ln a) a^x$
- 6) $\lim_{x \rightarrow \infty} e^x = \infty, \lim_{x \rightarrow -\infty} e^x = 0$
 $\lim_{x \rightarrow \infty} a^x = \infty, \lim_{x \rightarrow -\infty} a^x = 0$ if $a > 1$
 $\lim_{x \rightarrow \infty} a^x = 0, \lim_{x \rightarrow -\infty} a^x = \infty$ if $0 < a < 1$
- 7) The graph of 2^x is given below. The graph of a^x , for any $a > 1$, is similar.



Properties of Logarithms

In the following, x and y are arbitrary real numbers that are strictly bigger than 0, a is an arbitrary constant that is strictly bigger than one and e is 2.7182818284, to ten decimal places.

1) $e^{\ln x} = x$, $a^{\log_a x} = x$, $\log_e x = \ln x$, $\log_a x = \frac{\ln x}{\ln a}$

2) $\log_a (a^x) = x$, $\ln (e^x) = x$

$\ln 1 = 0$, $\log_a 1 = 0$

$\ln e = 1$, $\log_a a = 1$

3) $\ln(xy) = \ln x + \ln y$, $\log_a(xy) = \log_a x + \log_a y$

4) $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$, $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\ln\left(\frac{1}{y}\right) = -\ln y$, $\log_a\left(\frac{1}{y}\right) = -\log_a y$,

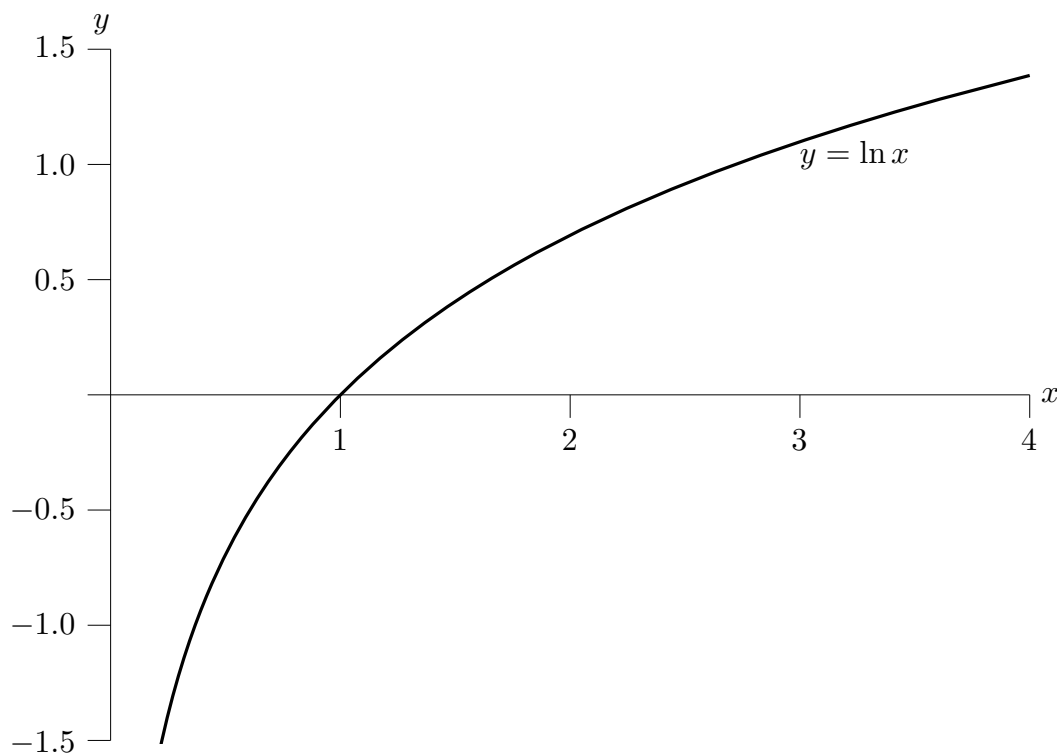
5) $\ln(x^y) = y \ln x$, $\log_a(x^y) = y \log_a x$

6) $\frac{d}{dx} \ln x = \frac{1}{x}$, $\frac{d}{dx} \ln(g(x)) = \frac{g'(x)}{g(x)}$, $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

7) $\lim_{x \rightarrow \infty} \ln x = \infty$, $\lim_{x \rightarrow 0} \ln x = -\infty$

$\lim_{x \rightarrow \infty} \log_a x = \infty$, $\lim_{x \rightarrow 0} \log_a x = -\infty$

8) The graph of $\ln x$ is given below. The graph of $\log_a x$, for any $a > 1$, is similar.



Taylor Expansions

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \dots + \frac{1}{n!}x^n + \dots$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots + \frac{(-1)^n}{(2n+1)!}x^{2n+1} + \dots$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \dots + \frac{(-1)^n}{(2n)!}x^{2n} + \dots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots + x^n + \dots$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots + (-1)^n x^n + \dots$$

$$\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \dots - \frac{1}{n}x^n - \dots$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots - \frac{(-1)^n}{n}x^n - \dots$$