Appendix 1

Excerpt from [BIR60, pp73-74]:

Before taking up the main business of the chapter, we pause briefly to make a few comments regarding three kinds of time derivatives used in the text. We might illustrate them with a homely example—namely the problem of reporting the concentration of fish in the Kickapoo River. Because the fish are moving, the fish concentration c will be a function of position (x, y, z) and time (t).

The Partial Time Derivative, $\partial c/\partial t$

Suppose we stand on a bridge and note how the concentration of fish just below us changes with time. We are observing then how the concentration changes with time at a *fixed* position in space. Hence by $\partial c/\partial t$ we mean the "partial of c with respect to t, holding x, y, z constant."

Total Time Derivative, dc/dt

Suppose now that instead of standing on the bridge we get in a motorboat and speed around on the river, sometimes going upstream, sometimes across the current, and perhaps sometimes downstream. If we report the change of fish concentration with respect to time, the numbers we report must also reflect the motion of the boat. The total time derivative is given by

$$\frac{dc}{dt} = \frac{\partial c}{\partial t} + \frac{\partial c}{\partial x}\frac{dx}{dt} + \frac{\partial c}{\partial y}\frac{dy}{dt} + \frac{\partial c}{\partial z}\frac{dz}{dt} \tag{3.0–1}$$

in which dx/dt, dy/dt, and dz/dt are the components of the velocity of the boat.

Substantial Time Derivative, Dc/Dt

Suppose that we get into a canoe, and, not feeling energetic, we simply float along counting fish. Now the velocity of the observer is just the same as the velocity of the stream v. When we report the change of fish concentration with respect to time, the numbers depend on the local stream velocity. This derivative is a special kind of total time derivative and is called the "substantial derivative" or sometimes (more logically) the "derivative following the motion." It is related to the partial time derivative as follows:

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + v_x\frac{\partial c}{\partial x} + v_y\frac{\partial c}{\partial y} + v_z\frac{\partial c}{\partial z} \tag{3.0–2}$$

in which v_x, v_y, and v_z are the components of the local fluid velocity v.

The reader should thoroughly master the physical meaning of these three derivatives. Remember that $\partial c/\partial t$ is the derivative at a fixed point in space and Dc/Dt is a derivative computed by an observer floating downstream with the fluid.