CHAPTER 3

ZEN and the ART OF SPACE-TIME KINETICS

The rest of this segment of this course will concentrate on solution methodology for space-time kinetic equations in multigroup form with delayed precursors, assuming diffusion instead of transport, as per equations 2-11 and 2-12, and assuming we know the coefficients.

We can write this more systematically as follows:

$$\frac{d\underline{\Psi}(\underline{r},t)}{dt} = \underline{\underline{A}}\,\underline{\Psi}(\underline{r},t)$$

where $$\underline{\Psi}(\underline{r},t) = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_G \\ C_1 \\ C_2 \\ \vdots \\ C_N \end{bmatrix}.$$

and

$$\underline{\underline{A}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1G} & a_{1,G+1} & \cdots & a_{1,G+N} \\ a_{21} & & & & & & & \\ \vdots & & a_{lj} & & & & & \\ a_{G1} & & & & a_{GG} & & & \\ a_{G+1,1} & & & & & a_{G+1,G+1} & & a_{G+1,G+N} \\ \vdots & & & & & & & \\ a_{G+N,1} & & & & a_{G+N,G} & a_{G+N,G+1} & & a_{G+N,G+N} \end{bmatrix}$$

Where

Upper left quadrant

$$\begin{cases} a_{ll} = V_l \left\{ \nu_l (1-\beta_l) \Sigma_{fl} \chi_{pl} + \Sigma_{sl} f_s (E_l \to E_l) - \Sigma_{rl} - \Sigma_{xl} \right. \\ \qquad \left. + \nabla \cdot D_l \nabla \right\} \qquad (l = j) \\ a_{lj} = V_l \left\{ \nu_j (1-\beta_j) \Sigma_{fj} \chi_{pl} + \Sigma_{sj} f_s (E_j \to E_l) \right\} \\ \qquad\qquad l \neq j \end{cases}$$

Upper right quadrant

$$\begin{cases} a_{l,G+i} = V_l \lambda_i C_i \chi'_{i,l} \\ l = 1 \text{ to } G \\ i = 1 \text{ to } N \end{cases}$$

lower left quadrant $\left\{ a_{G+i,j} = \beta_{ij} \gamma_j \varepsilon_{Fj} \right.$ $\quad i = 1 \rightarrow N$, $j = 1 \rightarrow G$

lower right quadrant $\begin{cases} a_{G+i,G+k} = 0 & \text{for } i \neq k \quad i = 1 \rightarrow N \\ a_{G+i,G+i} = -\lambda_i & \qquad\qquad\quad k = 1 \rightarrow N \end{cases}$

For 2 neutron groups (1 = fast, 2 = thermal) and 6 delayed precursor groups, and assuming

(1) no fast fissioning, $\Sigma_{f1} = 0$

(2) no upscatter, $f_s(E_2 \to E_1) = 0$

(3) all fission neutrons born in group 1, $\chi_{p2} = 0$,

we find:

$$a_{11} = V_1 \left\{ \Sigma_{s1} f_s(E_1 \to E_1) - (\Sigma_{a1} + \Sigma_{s1}) + \nabla \cdot D_1 \nabla \right\}$$

$$a_{22} = V_2 \left\{ -\Sigma'_{a2} + \nabla \cdot D_2 \nabla \right\}$$

$$a_{12} = V_1 \left\{ \nu_2 (1-\beta_1) \Sigma_{f2} \right\}$$

$$a_{21} = V_2 \left\{ \Sigma_{s1} f_s(E_1 \to E_2) \right\}$$

$$a_{1,6+i} = V_1 \lambda_i C_i \chi_{i1}$$

$$a_{2,6+i} = V_2 \lambda_i C_i \chi_{i2}$$

$$a_{2+i,1} = \beta'_{i1} \nu_1 \Sigma_{f1} = 0$$

$$a_{2+i,2} = \beta_{i2} \nu_2 \Sigma_{f2}$$

$$\left.\begin{aligned} a_{2+i,2+k} &= 0 \quad , i \neq k \\ a_{2+i,2+i} &= -\lambda_i C_i \end{aligned}\right\} \quad i, k = 1 \text{ to } 6$$

$$
\begin{bmatrix} \frac{\partial \phi_1}{\partial t} \\ \frac{\partial \phi_2}{\partial t} \\ \frac{\partial C_1}{\partial t} \\ \vdots \\ \frac{\partial C_6}{\partial t} \end{bmatrix}
=
\begin{bmatrix}
\overbrace{V_1\{\Sigma_{s1}f_s'' - \Sigma_{a1} - \Sigma_{s1} + \nabla\cdot D_1\nabla\}}^{\text{scattering, absorption, leakage + diffusion out}} & \overbrace{V_1\{\nu_2(1-\beta_1)\Sigma_{f2}\}}^{\text{fission source}} & V_1\lambda_1 C_1 \chi_{11} & V_1\lambda_2 C_2\chi_{21} & \cdots & V_1\lambda_6 C_6\chi_{61} \\
\underbrace{V_2\{\Sigma_{s1} f_s(E_1\to E_2)\}}_{\text{scattering down from fast (source)}} & \underbrace{V_2\{-\Sigma_{a2} + \nabla\cdot D_2\nabla\}}_{\text{absorption + diffusion sink 2}} & V_2\lambda_1 C_1\chi_{12} & V_2\lambda_2 C_2\chi_{22} & \cdots & V_2\lambda_6 C_6\chi_{62} \\
0 & \beta_{12}\nu_2\Sigma_{f2} & -\lambda_1 & 0 & \cdots & 0 \\
0 & \beta_{22}\nu_2\Sigma_{f2} & 0 & -\lambda_2 & & \\
\vdots & \vdots & & & \ddots & 0 \\
 & & & 0 & & \\
0 & \beta_{62}\nu_2\Sigma_{f2} & 0 & & & -\lambda_6
\end{bmatrix}
\begin{bmatrix} \phi_1 \\ \phi_2 \\ C_1 \\ \vdots \\ C_6 \end{bmatrix}
$$

6 × 6

delayed neutrons (source)

production of precursors

decayed precursors

Typical values for D_2O moderated give

$$A = \begin{bmatrix} -7\times10^{8} & & & & & & & \\ +10^{6} & -6 & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \end{bmatrix}$$

Typical values for fresh U^{235}/U^{238} Fuel give

$$A = \begin{bmatrix} -15\times10^{8} & +.5\times10^{8} & & & & & & \\ +7.32\times10^{4} & -7.00\times10^{4} & & & & & & \\ & & (\beta\nu)(\Sigma_{2F}) & & & & & \\ & & (.052)(.2) & -.0124 & & & & \\ & & (.346)(.2) & & -.0305 & & & \\ & & (.310)(.2) & & & -.111 & & \\ & & (.0624)(.2) & & & & -.301 & \\ & & (.182)(.2) & & & & & -1.14 \\ & & (.066)(.2) & & & & & & -3.01 \end{bmatrix}$$

As you can see, the matrix is often very sparse and (the size of the coefficients vary over many orders of magnitude). As we shall see, one can always write a program that will grind away. However, if we're clever we can make use of some dominant characteristics to speed up the solution procedure. Some obvious ways to speed up the solution procedure are:

1) If you are only interested in the speedier parts of the solution, replace the slower parts with their equilibrium value or any known values. An example is replacing the delayed precursor equations with their equilibrium or fresh fuel concentrations and substituting these values into the flux equations directly. These concentrations can be updated periodically. In this way the long elapsed time simulations using small Δt's are eliminated.

2) If you are only interested in the long term parts of the solution, say the delayed precursors, then force ϕ = constant (by reactivity control) and enter these ϕ values directly into the precursor equations. In this way the small Δt required to track the ϕ is no longer necessary.

Beware in mind that this matrix A, and the vector $\underline{\psi}$, are functions of space (and the matrix can be stiff in space if one dimension is much larger than another). So a matrix must be set up for each grid point chosen. A typical 3-D grid might be 100×100×100 points (10^6 points) and at each point G+N equations are to be solved for each time step. This is expensive and this is the reason why a lot of attention has been given to finding efficient algorithms for space-time kinetics problems.

Aside: A Design Approach

It generally pays handsomely to adopt the approach that you can get 80% of the answer with 20% of the effort by roughing in the problem first and determining the main characters in your particular problem. In industry, this rough answer is often sufficient in providing ballpark figures for "executive decisions". For instance, if the boss needs to know the rough physical

dimension, for a reactor core, you can probably make do with solving the steady state equations (with delayed precursors at equilibrium levels) with one or two neutron groups and rough estimates of the basic parameters.

If the boss is interested in flux tilts, then perhaps a Laplace transform to state space would be the better line of attack to quickly check out the key parameters which affect stability, and so on.

This approach is preferred even if the cost of running the full-blown equation set was not high. This is because it is simply good design practice to look before you leap, ie, get the feel of the result first. In this way you develop intuition. The solution of the full-blown equation set in all its glory gives precise numerical answers. Often, you can get less precise but more accurate answers by following the above design procedure.

You will find that managers are typically skeptical of "computer jockeys" who quote precise answers. And rightly so. Equally, the jockey is equally skeptical of the "cowboy manager" who rides by the seat of his pants. And rightly so.

What is needed is an informed balance between the two approaches. As strange as it may seem, the number of people in the industry who manage to strike this balance is small. How you affect this balance is up to you.

POINTS TO CONTEMPLATE

1) The basic equation set in matrix form is

$$\frac{d\underline{\Psi}}{dt} = \underline{\underline{A}}\,\underline{\Psi}.$$

Compare this to $\frac{dx}{dt} = Ax \Rightarrow x = x_0 e^{At}$.

What does this suggest? Can we use this somehow?

2) Are the basic equations bounded & unbounded?

What does this mean, practically?

References

none.