CHAPTER 2

GENELOGY OF APPROXIMATIONS

The Boltzmann Transport Eqn (1-13) and the delayed precursor eqn (1-14) require simplification to be solved. Transport theory work (used for some shielding calculations, for instance) generally expresses the precursor eqn. and employs various methods like P_L or S_N approximations to the space-energy-angle dependance. Time is usually dropped (ie steady state). For this course, we ignore most of the branches of the family tree and deal with the diffusion approximation lineage only. This is the world of space-time kinetics.

Figure 2-1 outlines the many approximations that can be made within the diffusion approximation.

Space-time kinetics generally refers to the coupled space and time variation of the neutron flux (or density) for large power reactors. This is so because for large high flux cores, the space variation is not as time independent as for the tightly coupled (spatially) smaller cores. For small cores, leakage at the boundaries dominate the flux shape. Not so with larger cores. Also, for high fluxes, the precursor

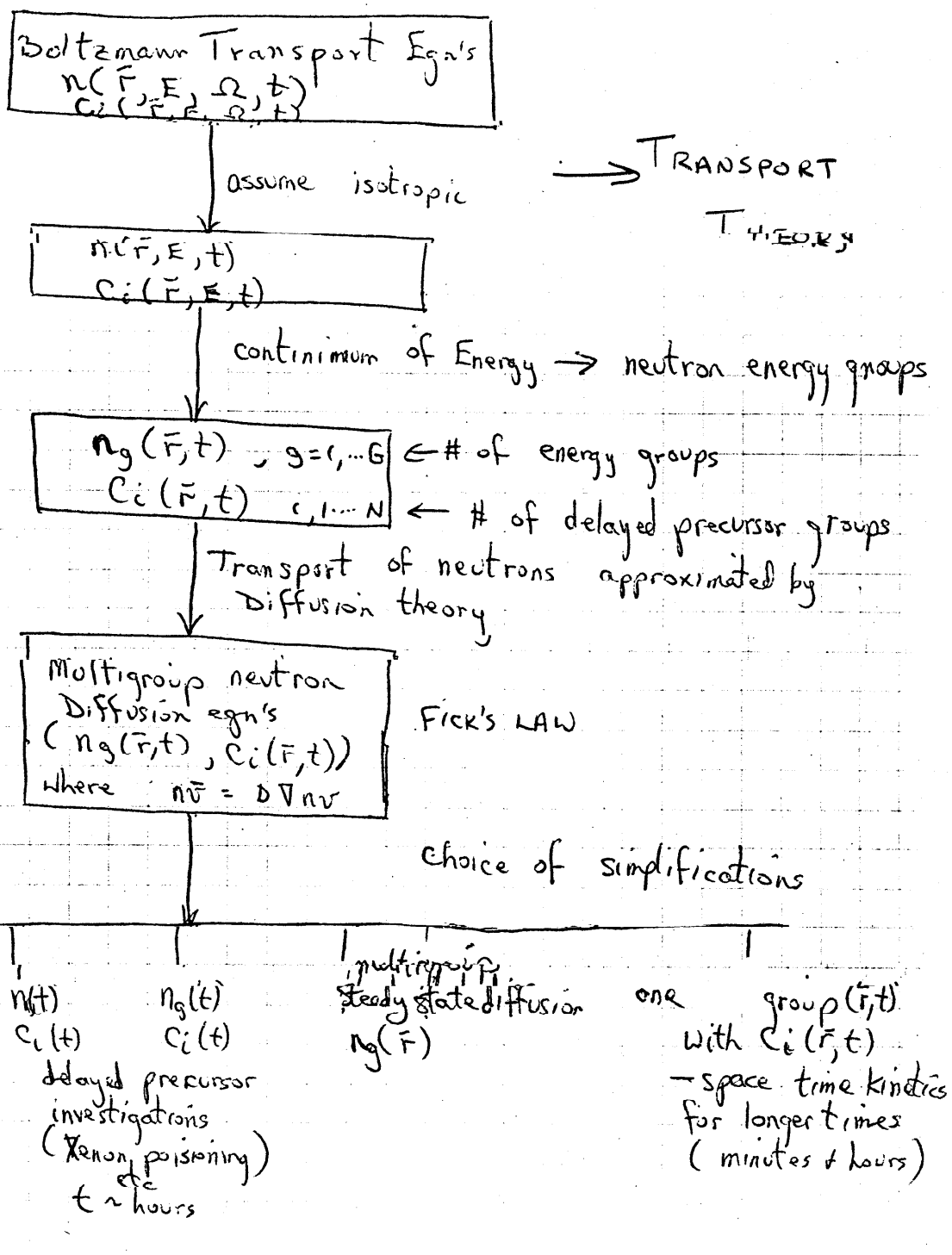

Boltzmann Transport Eqn's
$n(\bar{r}, E, \Omega, t)$
$C_i(\bar{r}, E, \Omega, t)$
assume isotropic
TRANSPORT THEORY
$n(\bar{r}, E, t)$
$C_i(\bar{r}, E, t)$
continuum of Energy → neutron energy groups
$n_g(\bar{r}, t)$, $g = 1, \ldots G$ ← # of energy groups
$C_i(\bar{r}, t)$ $i = 1 \ldots N$ ← # of delayed precursor groups
Transport of neutrons approximated by Diffusion theory
Multigroup neutron Diffusion eqn's
$(n_g(\bar{r}, t), C_i(\bar{r}, t))$
where $n\bar{v} = D \nabla n v$
FICK'S LAW
Choice of simplifications
Multigroup $(\bar{r}, t)$
No C_i
(space-time Kinetics)
For short times (C_i ~ const)
t ~ seconds
$n_g(t)$
$C_i(t)$
delayed precursor investigations
(Xenon poisoning etc)
t ~ hours
multigroup steady state diffusion
$n_g(\bar{r})$
one group $(\bar{r}, t)$
with $C_i(\bar{r}, t)$
– space time kinetics for longer times
(minutes & hours)

FIGURE 2-1

on poison concentration vary with flux and time, giving a complex interaction.

<u>No Source Approximation</u> Since we are generally dealing with operating reactors at or near criticality, we can safely assume that

$$S_{misc} = 0. \qquad 2\text{-}1$$

We can always bring this term back if need be.

<u>Isotropic Approximation</u> We also are considering, in this course, nuclear events below ~2 MeV. Thus the nucl events can be considered isotropic, ie no $\underline{\Omega}$ dependence.

Thus

$$\int_{\underline{\Omega}'} f(\underline{r}; E', \underline{\Omega}' \rightarrow E, \underline{\Omega}; t)\, d\underline{\Omega}' \qquad 2\text{-}2$$

$$= f(\underline{r}; E' \rightarrow E; t)$$

where the dimensions of $f(r; E' \rightarrow E; t)$ are, obviously, not the same as for $f(\underline{r}; E', \underline{\Omega}' \rightarrow E, \underline{\Omega}; t)$.

Similarly:

$$\int_{\underline{\Omega}'} n(\underline{r}, E', \underline{\Omega}', t)\, d\underline{\Omega} = n(\underline{r}, E, t), \qquad 2\text{-}3$$

etc.

If we drop the implicit r, t parametric dependence (for clarity) in our notation, we have:

$$\frac{\partial n(E)}{\partial t} = \int_{E'} v(E') n(E') \Big\{ \nu(E)(1-\beta(E'))\Sigma_f(E')\chi_p(E) + \Sigma_s(E')f_s(E' \to E) \Big\} dE'$$

$$- \nabla \cdot n(E)\,\underline{v}(E) + \sum_{i=1}^{N} \lambda_i \chi_i(E) C_i - v(E) n(E) \Big(\Sigma_a(E) + \Sigma_s(E$$

2-4

where all parameters except, v, λ_i and χ_i have an r, t dependence.

Also:

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \int_{E'} \beta_i(E') \nu(E') v(E') n(E') \Sigma_f(E')\, dE'$$

2-5

DIFFUSION APPROXIMATION

By analogy to heat and mass diffusion:

$$n(\underline{r}, E, t)\,\underline{v}(E) = -D(\underline{r}, E, t) \nabla \Big\{ n(\underline{r}, E, t) v(E) \Big\}$$

$$\equiv -D(\underline{r}, E, t) \nabla \phi(\underline{r}, E, t).$$

2-6

We now recast the equations in terms of flux, ϕ, using 2-6:

$$\frac{1}{V(E)}\frac{\partial \phi(E)}{\partial t} = \int_{E'} \phi(E')\left\{\nu(E')[1-\beta(E')]\Sigma_f(E')\chi_p(E) + \Sigma_s(E')f_s(E'\to E)\right\}dE'$$

$$+ \nabla\cdot\left(D(E)\nabla\phi(E)\right) + \sum_{i=1}^{N}\lambda_i\chi_i(E)C_i - \phi(E)\left(\Sigma_a(E)+\Sigma_s(E)\right)$$

2-7

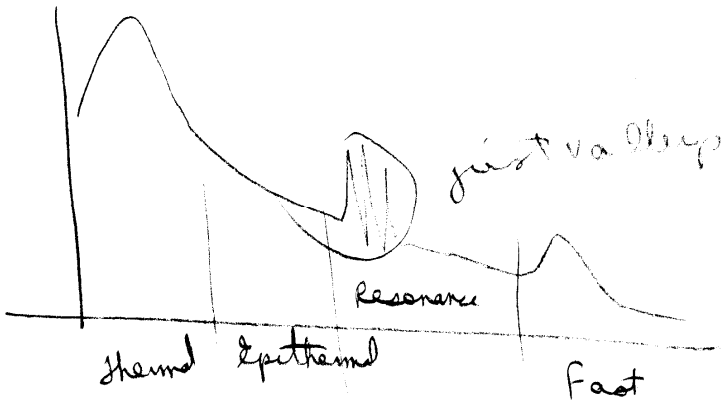
The precursor equation is:

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \int_{E'}\left(\beta_i(E')\nu(E')\phi(E')\Sigma_f(E')\right)dE'.$$

2-8

MULTIGROUP APPROXIMATION

The above equations are still intractable. There are a number of ways to deal with the energy dependence. One successful and flexible way is to divide the energy continuum up into G groups. Conventionally the split is as follows:



For CANDU's, the neutron spectrum is well thermalized and a two group approximation has proved successful, one group for the thermal neutrons and one group for all else.

If we define:

$$\psi_\ell \equiv \int_{\Delta E_\ell} \psi(E)\, dE \qquad 2-9$$

and hence $\sum_{j=1}^{G} \psi_j = \int_E \psi(E')\, dE'$ 2-10

we get:

$$\frac{1}{v_\ell}\frac{\partial \phi_\ell}{\partial t} = \sum_{j=1}^{G} \phi_j \left\{ \nu_j (1-\beta_j) \Sigma_{fj} \chi_{p\ell} + \Sigma_{sj} f_s(E_j \to E_\ell) \right\} + \nabla \cdot D_\ell \nabla \phi_\ell + \sum_{i=1}^{N} \lambda_i c_i \chi_{i\ell} - (\Sigma_{a\ell} + \Sigma_{s\ell})\phi_\ell \qquad 2-11$$

and

$$\frac{\partial c_k}{\partial t} = -\lambda_k c_k + \sum_{j=1}^{G} \beta_{kj} \nu_j \Sigma_{fj} \phi_j \qquad 2-12$$

where the r,t dependences are implicit.

Note that the coefficients are obtained by the integration of the respective terms over the energy interval. For instance:

$$\Sigma_\ell \phi_\ell = \int_{\Delta E_\ell} \Sigma_a(E)\, \phi(E)\, dE, \qquad 2-13$$

$$\nabla \cdot D_\ell \nabla \phi_\ell = \int_{\Delta E_\ell} \nabla \cdot D(E) \nabla \phi(E)\, dE, \qquad 2-14$$

etc.

Thus, we really need to know the flux in order to generate the coefficients that we need to calculate the flux! This leads to iteration or experimental work.

STEADY STATE

This is rather obvious; just set the left hand side of 2-11 and 2-12 to zero.

SHORT TIMES

The time constant for the delayed precursor concentration change is of the order of minutes to hours while the neutron flux changes with a time constant of seconds. For a great deal of work, it is sufficient to assume that the precursor concentration is constant or:

$$\frac{\partial C_i}{\partial t} \simeq 0 . \qquad 2\text{-}15$$

Thus:

$$\lambda_i C_i = \sum_{j=1}^{G} \beta_{ij} \nu_j \Sigma_{fj} \phi_j \qquad 2\text{-}16$$

Substituting this into 2-11 gives:

$$\frac{1}{v_\ell}\frac{\partial \phi_\ell}{\partial t} = \sum_{j=1}^{G} \phi_j \left\{ \nu_j (1-\beta_j) \Sigma_{fj} \chi_{pl} + \Sigma_{sj} f_s(E_j \to E_\ell) \right\}$$
$$+ \nabla \cdot D_\ell \nabla \phi_\ell + \sum_{i=1}^{N} \sum_{j=1}^{G} \beta_{ij} \nu_j \chi_{i\ell} \Sigma_{fj} \phi_j - (\Sigma_{a\ell} + \Sigma_{s\ell}) \phi_\ell$$

$$= \sum_{j=1}^{G} \phi_j \left\{ \nu_j \chi_{j\ell} \Sigma_{fj} + \Sigma_{sj} f_s(E_j \to E_\ell) \right\}$$
$$+ \nabla \cdot D_\ell \nabla \phi_\ell - (\Sigma_{a\ell} + \Sigma_{s\ell}) \phi_\ell \qquad 2\text{-}17$$

where $\chi_{j\ell} \equiv (1-\beta_j)\chi_{p\ell} + \sum_{i=1}^{N} \beta_{ij} \chi_{i\ell}$.

Thus we have the flux equation in terms of flux and coefficients only.

SPACE INDEPENDENCE — POINT KINETICS

If there is no space dependence, simply set $\nabla \cdot D_\ell \nabla \phi_\ell$ to zero. Sometimes, this buckling term is known and fixed. In that case, the value can be entered as a parameter in the equation. Equations 2-11 and 2-12 in the space independent form are the point kinetics equations for G groups. Rewritten in more familiar terms for the two simple case of 1 group:

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = \left[(1-\beta)k_\infty - 1\right]\Sigma_a \phi + p\sum_i \lambda_i C_i \qquad 2\text{-}18$$

$$\frac{\partial C_i}{\partial t} = \beta_i \frac{k_\infty}{p}\Sigma_a \phi - \lambda_i C_i \qquad 2\text{-}19$$

where $G = 1 \Rightarrow f_s = 1$

$$\nu(1-\beta)\Sigma_f \chi_p = (1-\beta)k_\infty \Sigma_a = (1-\beta)\epsilon\eta p f \Sigma_a$$

$$\chi_{i\ell} = p_i$$

Equations 2-18 and 2-19 are solvable as you've seen in the past and as you'll see again soon.

POINTS TO CONTEMPLATE

1) You might try to anticipate what solution procedures would be appropriate for each approximation.

2) Try to express coefficients like $\bar{p}$, k_∞, etc in fundamental terms, ie flux weighted averages, etc.

3) The diffusion approximation also drops out neatly from the P_1 approximation in Transport theory. Try it. What does it mean physically?

Reference: Name.