CHAPTER 1

NEUTRON TRANSPORT EQUATION
(THE MOTHER LODE)

A neutron density, $n = n(\underline{r}, E, \underline{\Omega}, t)$, is, in general, a function of space, energy, angle and time. It would exist, in general, in a heterogeneous environment where the material properties are also a function of $\underline{r}$, E, $\underline{\Omega}$ and t. We treat the neutron population as a continuum, avoiding statistical fluctuations. Typically, $\phi \equiv nv \sim 10^{14}$ neutrons/cm^2-sec, therefore, the statistical fluctuations are negligible. Also we ignore neutron-neutron interations since the neutron density is small compared to the media density ($\sim 10^{22}$ atoms/cm^3.)..

The continuity or conservation equation, based on our intuitive experience, states:

$$\frac{d}{dt}\int_{V} n \, dV = \int_{V} \sum_i S_i \, dV \qquad 1-1$$

or the substantial derivative of the neutron population in a volume, V = sum of sinks and sources.

The Reynold's Transport Theorem states:

$$\frac{d}{dt}\int_{\forall} \psi \, d\forall = \int_{\forall} \frac{\partial \psi}{\partial t} d\forall + \int_{s} \psi \, \underline{v} \cdot d\underline{s} \qquad 1-2$$

(TOTAL) = (GENERATION) + (FLUX)

where ψ is any field parameter

$\underline{V}$ = velocity of the parameter ($\underline{V} = V\,\underline{\Omega}$)

$\underline{S}$ = normal surface vector.
$\forall$ = volume.

Thus:

$$\int_{\forall} \frac{\partial n}{\partial t} d\forall = -\int_{s} n\underline{V}\cdot d\underline{s} + \int_{\forall} \sum_i S_i \, d\forall . \qquad 1-3$$

Using Gauss' divergence theorem:

$$\int_{s} \underline{A}\cdot d\underline{S} = \int_{\forall} \nabla \cdot A \, d\forall \qquad 1-4$$

gives:

$$\int_{\forall} \frac{\partial n}{\partial t} d\forall = -\int_{\forall} \nabla \cdot (n\underline{V}) \, d\forall + \int_{\forall} \sum_i S_i \, d\forall , \qquad 1-5$$

or:

$$\frac{\partial n}{\partial t} = -\nabla \cdot n\underline{V} + \sum_i S_i . \qquad 1-6$$

where $n = n(\underline{r}, E, \underline{\Omega}, t)$

$V = V(E)$, $\underline{V} = V\,\underline{\Omega}$

$S_i = S_i(\underline{r}, E, \underline{\Omega}, t)$

We further limit ourselves to the following sinks and sources:

1) fission source, prompt.
2) scattering into $(\underline{r}, E, \underline{\Omega}, t)$.
3) delayed precursor source
4) loss out of $(\underline{r}, E, \underline{\Omega}, t)$ by absorption or scattering.
5) misc. sources.

Fission Source

To create a neutron at some $\underline{r}, E, \underline{\Omega}, t$ from neutrons at $\underline{r}', E', \underline{\Omega}', t'$, the n' neutrons are absorbed at the ' conditions, causing fission neutrons to be created. The process is a function of both ~~the~~ primed and unprimed conditions. ~~Hence~~.

$$S_{fission} = \int_{E}\int_{\underline{\Omega}'} v' n' \quad \nu' (1-\beta') \Sigma_f' \chi_p \quad d\underline{\Omega}' \, dE' \qquad 1-7$$

where $v' = v(E')$ = velocity
$n' = n(r, E', \underline{\Omega}', t)$ = neutron density
$\nu' = \nu(r, E', \underline{\Omega}', t)$ = total # produced per fission in the fuel caused by a neutron of energy E', direction $\underline{\Omega}'$
$\beta' = \beta(\underline{r}, E', \underline{\Omega}', t) = \sum_i \beta_i$ = delayed fraction.
$\Sigma_f' = \Sigma_f(\underline{r}, E', \underline{\Omega}', t)$ = fission crossection
$\chi_p = \chi_p(\underline{r}, E, \underline{\Omega}, t)$ = prompt neutron spectrum.

By integrating over E' + Ω', we get the total fission contribution of fission neutrons ending up in the interval about E + Ω.

Things to note:

1) $\underline{r}' = \underline{r}$ because a fission creates neutrons "on the spot".

2) $t' = t$ because the time delay is very small ($\sim 10^{-14}$ sec.).

3) The product $\iint_{E'\,\underline{\Omega}'} v'n' \, \nu'(1-\beta')\Sigma_f' \, dE' d\underline{\Omega}'$ gives the total number of fission neutrons produced per second. The spectrum factor, χ_p, proportions this total to the respective energy group and direction group.

4) The units of $n(\underline{r}, E, \underline{\Omega}, t)$ are neutrons per unit space per unit energy, etc, ie:

$$\#/(\text{cm}^3\text{-eV-steradian-sec}).$$

SCATTERING SOURCE

Neutrons at $E', \underline{\Omega}'$ are scattered out of $E', \underline{\Omega}'$ into $E, \underline{\Omega}$ (really the interval $dE'd\underline{\Omega}'$ about $E', \underline{\Omega}'$, etc), according to the probability density function $f_s(r; E', \underline{\Omega}' \rightarrow E, \underline{\Omega}; t)$ and the normal macroscopic crosssection $\Sigma_s(\underline{r}, E', \underline{\Omega}', t)$.

Thus:

$$S_{SCATTER, 'IN'} = \int_{E'} \int_{\Omega'} v(E')\, n(\underline{r}, E', \underline{\Omega}', t)\, \Sigma_s(\underline{r}, E', \underline{\Omega}', t)\, f_s(\underline{r}; E', \underline{\Omega}' \rightarrow E, \underline{\Omega}; t)\, dE'\, d\underline{\Omega}'. \qquad \leftarrow 1-8$$

DELAYED SOURCE

The delayed precursor source is as you've seen before except that the precursor, $C_i(\underline{r}, t)$, produces a spectrum of neutron energies :- $\chi_i(E, \Omega)$. Thus:

$$S_{PRECURSOR} = \sum_{i=1}^{N} \lambda_i\, \chi_i(E, \underline{\Omega})\, C_i(\underline{r}, t). \qquad 1-9$$

Loss Terms

Loss by absorption and scattering follow directly as:

$$S_{ABSORPTION} = -V(E)\, n(\underline{r}, E, \underline{\Omega}, t)\, \Sigma_a(\underline{r}, E, \underline{\Omega}, t) \qquad 1\text{-}10$$

$$S_{SCATTER\ OUT} = -V(E)\, n(\underline{r}, E, \underline{\Omega}, t)\, \Sigma_s(\underline{r}, E, \underline{\Omega}, t)\,. \qquad 1\text{-}11$$

Misc Source

For completeness we write:

$$S_{MISC} = S(\underline{r}, E, \underline{\Omega}, t)\,. \qquad 1\text{-}12$$

Summing Up

Summing these up and plugging into 1-6 gives:

$$\frac{\partial n}{\partial t} = \int_{E'}\int_{\Omega'} V' n' \left\{ \nu'(1-\beta')\Sigma_f' \chi_p + \Sigma_s' f_s' \right\} d\underline{\Omega}'\, dE' \qquad 1\text{-}13$$

$$- \nabla \cdot n\underline{V} + \sum_{i=1}^{N} \lambda_i \chi_i C_i - Vn(\Sigma_a + \Sigma_s)$$

(Boltzmann Transport Eqn)

$$+ S_{MISC}$$

Similarly the delayed precursor concentration equation is:

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \int_{E'}\int_{\Omega'} (\beta_i' \nu' V' n' \Sigma_f')\, d\underline{\Omega}'\, dE'\,. \qquad 1\text{-}14$$

Although these equations are very general and unsolvable in practice, they embody several assumptions, as already noted. We'll have to make even more to get to the stage where we can solve these equations.

Assumptions so far:

1) $t' = t$

2) $r' = r$

3 no statistical fluctuations

4 no other sinks or sources than those listed.

5 no neutron - neutron interactions.

THINGS TO CONTEMPLATE

1) Can you find any other assumptions ?

2) This continuity equation is similar to the conservation of mass in thermalhydraulics. Why do we not also generally consider the conservation of momentum and energy ?

3) Expand on the implications of $t' = t$, $r' = r$.

Reference: 1) See Appendix 1, Section 1.1 (handout)

2 See Appendix 2, Reynolds Transport Theorem.

3) See any of the classics, like Bell & Glasstone

Re Point 2

Conservation of momentum and energy used on a neutron-nucleus interaction level. Recoils, etc, lead to cross sections which are input into the neutron balance equation. Compare this to fluid mechanics. The neutrons don't interact with each other but fluid particles do. Hence the fluid mass, energy + momentum equations are tightly linked. The neutrons ~~only~~ affect each other only via temperature changes, etc, brought about by fissioning, whereas, fluid particles shear off each other and with the walls. This is a fundamental difference.