APPENDIX 1

DERIVATION OF THE BASIC NEUTRON EQUATIONS

A.1.1 THE GENERAL EQUATIONS

The general neutron transport equations (16,74) can be written as follows:

$$\frac{\partial n(\underline{r},E,\underline{\Omega},t)}{\partial t} = \int_{E'} \int_{\underline{\Omega}'} v(E')\, n(\underline{r},E',\underline{\Omega},t)\{\nu(\underline{r},E',\underline{\Omega}',t)$$

$$[1 - \beta(\underline{r},E',\underline{\Omega}',t)]\, \Sigma_f(\underline{r},E',\underline{\Omega}',t)\, \chi_p(\underline{r},E,\underline{\Omega},t)$$

$$+ \Sigma_s(\underline{r},E',\underline{\Omega}',t)\, f_s(\underline{r}:E',\underline{\Omega}' \rightarrow E,\underline{\Omega}:t)\}\, d\underline{\Omega}'dE'$$

$$- \nabla[n(\underline{r},E,\underline{\Omega},t)\ .\ \underline{v}(E)] + \sum_{i=1}^{N} \lambda_i\, \chi_i(E,\underline{\Omega})\, C_i(\underline{r},t)$$

$$+ s(\underline{r},E,\underline{\Omega},t) - [\Sigma_a(\underline{r},E,\underline{\Omega},t) + \Sigma_s(\underline{r},E,\underline{\Omega},t)]$$

$$v(E)\, n(\underline{r},E,\underline{\Omega},t) \qquad \text{(A1.1)}$$

and

$$\frac{\partial C_i(\underline{r},t)}{\partial t} = -\lambda_i\, C_i(\underline{r},t) + \int_{E'}\int_{\underline{\Omega}'} \beta_i(\underline{r},E',\underline{\Omega}'t)\, \nu(\underline{r},E',\underline{\Omega}',t)$$

$$v(E')\, \Sigma_f(\underline{r},E',\underline{\Omega}',t)\, n(\underline{r},E',\underline{\Omega}',t)\, d\underline{\Omega}'dE' \quad , \qquad \text{(A1.2)}$$

where

t = time [s] ,

E = energy [Mev] ,

$\underline{\Omega}$ = direction (vector) [radian] ,

$v(E)$ = velocity [cm/s] ,

$\underline{v}(E)$ = velocity (vector) = $v(E)\ \underline{\Omega}$ [cm/s] ,

$\underline{r}$ = space (vector) [cm] ,

$n(\underline{r},E,\underline{\Omega},t)$ = neutron density [neutrons/cm^3 radian Mev] ,

$\nu(\underline{r},E,\underline{\Omega},t)$ = total number of neutrons produced per fission in the fuel caused by a neutron of energy E and direction $\underline{\Omega}$ [dimensionless] ,

$\Sigma(\underline{r},E,\underline{\Omega},t)$ = macroscopic cross section [cm^{-1}] ,

$f_s(\underline{r};E',\underline{\Omega}' \rightarrow E,\underline{\Omega};t)$ = probability density function for a neutron at $(E',\underline{\Omega}')$ scattering to $(E,\underline{\Omega})$ [radian^{-1} Mev^{-1}] ,

$s(\underline{r},E,\underline{\Omega},t)$ = extraneous neutron sources [neutrons/cm^3 radian Mev s] ,

$C_i(\underline{r},t)$ = i^{th} delayed precursor concentration, $i = 1,2\ldots N$ [number/cm^3] ,

λ_i = decay constant for i^{th} delayed precursor [s^{-1}] ,

β_i = fraction of total fission production from i^{th} delayed precursor [dimensionless] ,

$$\beta = \sum_{i=1}^{N} \beta_i \quad ,$$

$\chi_p(\underline{r},E\ \underline{\Omega},t)$ = normalized fission spectrum [radian^{-1} Mev^{-1}] ,

$\chi_i(E,\underline{\Omega})$ = normalized spectrum for neutrons emitted by i^{th} delayed precursor [radian^{-1} Mev^{-1}] ,

with subscripts:

f = fission event,

s = elastic and inelastic scattering event,

a = absorption event.

Equation (A1.1) can be expressed in words as follows. The net rate of accumulation of neutrons, given by the left hand side of equation (A1.1), is composed of several sinks and sources. From left to right, the first term on the right hand side is the rate of gain in neutron population, at the space volume element about $(\underline{r},E,\underline{\Omega},t)$ from the element about $(\underline{r},E',\underline{\Omega}'t)$, by prompt fissioning and scattering. The second term represents the net rate of loss by neutron transport out of the volume element about $\underline{r}$. The third term is the delayed neutron source from the delayed precursors. The fourth is that due to extraneous sources, and the last is the rate of loss by absorption and scattering. The balance condition for the delayed precursors, equation (A1.2) is simpler in that there is no physical movement of the precursors and there is only one sink, radioactive decay, and one source, fission fragment production due to the fissioning process.

These equations hold in general except when there is material transport, as in a reactor disassembly or in aqueous solution reactors, or when the neutron population is so low as to have significant statistical fluctuations. These are rare cases, certainly not applicable for this study.

A1.2 SIMPLIFYING ASSUMPTIONS

A number of simplifying assumptions can be made to transform the general transport equations into a tractable formalism. We may set $s = 0$ since there are no extraneous sources in operating CANDU reactors. Also, for nuclear events below 2 Mev., typical of nuclear reactors, the production, scattering and absorption events can be considered isotropic, that is, independent of direction. Thus,

$$\int_{\underline{\Omega}'} f(\underline{r};E',\underline{\Omega}' \rightarrow E,\underline{\Omega};t)\, d\underline{\Omega}' = f(\underline{r};E' \rightarrow E;t) \quad , \tag{A1.3}$$

$$\int_{\underline{\Omega}'} n(\underline{r},E',\underline{\Omega}',t)\, d\underline{\Omega}' = n(\underline{r},E',t) \quad , \tag{A1.4}$$

etc.

Hence,

$$\frac{\partial C_i(\underline{r},t)}{\partial t} = - \lambda_i\, C_i(\underline{r},t) + \int_{E'} \beta_i(\underline{r},E',t)\, \nu(\underline{r},E',t)\, v(E')\, \Sigma_f(\underline{r},E',t)\, n(\underline{r},E',t)\, dE'$$

and

$$\begin{aligned}\frac{\partial n(\underline{r},E,t)}{\partial t} = &\int_{E'} v(E')\, n(\underline{r},E't)\{\nu(\underline{r},E',t)[1 - \beta(\underline{r},E',t)] \\ &\Sigma_f(\underline{r},E',t)\, \chi_p(\underline{r},E,t) + \Sigma_s(\underline{r},E',t)\, f_s(\underline{r};E' \rightarrow E;t)\}dE' \\ &- \nabla[n(\underline{r},E,t) \cdot \underline{v}(E)] + \sum_{i=1}^{N} \lambda_i\, \chi_i(E)\, C_i(\underline{r},t) \\ &- [\Sigma_a(\underline{r},E,t) + \Sigma_s(\underline{r},E,t)]\, v(E)\, n(\underline{r},E,t) \quad .\end{aligned} \tag{A1.6}$$

Since the velocity vector is space independent

$$\nabla \cdot [n(\underline{r},E,t)\, \underline{v}(E)] = \nabla[n(\underline{r},E,t) \cdot \underline{v}(E)] \tag{A1.7}$$

and, by analogy to heat and mass diffusion,

$$n(\underline{r},E,t)\ \underline{v}(E) = -D(\underline{r},E,t)\ \nabla[n(\underline{r},E,t)\ v(E)]$$

$$\equiv -D(\underline{r},E,t)\ \nabla\phi(\underline{r},E,t) \quad . \tag{A1.8}$$

Equation (A1.6) is, then,

$$\frac{\partial n(\underline{r},E,t)}{\partial t} = \int_{E'} v(E')\ n(\underline{r},E',t)\{\nu(\underline{r},E',t)[1-\beta(\underline{r},E',t)]$$

$$\Sigma_f(\underline{r},E',t)\ \chi_p(\underline{r},E,t) + \Sigma_s(\underline{r},E',t)\ f_s(\underline{r};E' \rightarrow E;t)\}dE'$$

$$+ \nabla \cdot D(\underline{r},E,t)\ \nabla[n(\underline{r},E,t)\ v(E)] + \sum_{i=1}^{N} \lambda_i\ \chi_i(E)\ C_i(\underline{r},t)$$

$$- [\Sigma_a(\underline{r},E,t) + \Sigma_s(\underline{r},E,t)]\ v(E)\ n(\underline{r},E,t) \quad . \tag{A1.9}$$

The multigroup approximation is obtained by discretizing the energy continuum. Denoting the ℓ^{th} group by the subscript, ($\ell = 1,2,\ldots G$), and integrating equation (A1.9) over the energy interval, ΔE_ℓ, gives

$$\frac{\partial}{\partial t}\int_{\Delta E_\ell} n(\underline{r},E,t)dE = \int_{E'} v(E')\ n(\underline{r},E',t)[\int_{\Delta E_\ell}\{\nu(\underline{r},E',t)$$

$$[1-\beta(\underline{r},E',t)]\ \Sigma_f(\underline{r},E',t)\ \chi_p(\underline{r},E,t)$$

$$+ \Sigma_s(\underline{r},E',t)\ f_s(\underline{r};E' \rightarrow E;t)\}dE]dE'$$

$$+ \int_{\Delta E_\ell} \nabla \cdot D(\underline{r},E,t)\ \nabla[n(\underline{r},E,t)\ v(E)]dE$$

$$+ \sum_{i=1}^{N} \lambda_i\ C_i(\underline{r},t) \int_{\Delta E_\ell} \chi_i(E)dE$$

$$- \int_{\Delta E_\ell} [\Sigma_a(\underline{r},E,t) + \Sigma_s(\underline{r},E,t)]\ v(E)\ n(\underline{r},E,t)dE \quad . \tag{A1.10}$$

We now introduce the following notation for conciseness and clarity:

$$\sum_{j=1}^{G} \int_{\Delta E_j} dE' \equiv \int_{E'} dE' \tag{A1.11}$$

and

$$T_\ell(\underline{r},\ldots) \equiv \int_{\Delta E_\ell} T(E,\underline{r},\ldots)dE \quad , \tag{A1.12}$$

where T is a general function. Equation (A1.10) becomes

$$\frac{\partial\, n_\ell(\underline{r},t)}{\partial t} = \sum_{j=1}^{G} v_j\, n_j(\underline{r},t)\{\nu_j(\underline{r},t)[1 - \beta_j(\underline{r},t)]\, \Sigma_{fj}(\underline{r},t)\, \chi_{p\ell}(\underline{r},t)$$

$$+ \Sigma_{sj}(\underline{r},t)\, f_s(\underline{r};E_j \to E_\ell;t)\} + \nabla \cdot D_\ell(\underline{r},t)\, \nabla[n_\ell(\underline{r},t)\, v_\ell]$$

$$+ \sum_{i=1}^{N} \lambda_i\, C_i(\underline{r},t)\, \chi_{i\ell}$$

$$- [\Sigma_{a\ell}(\underline{r},t) + \Sigma_{s\ell}(\underline{r},t)]\, v_\ell\, n_\ell(\underline{r},t) \quad . \tag{A1.13}$$

The delayed precursor equation becomes, through a similar analysis,

$$\frac{\partial\, C_i(\underline{r},t)}{\partial t} = -\lambda_i\, C_i(\underline{r},t) + \sum_{j=1}^{G} \beta_{ij}(\underline{r},t)\, \nu_j(\underline{r},t)\; v_j\, \Sigma_{fj}(\underline{r},t)$$

$$n_j(\underline{r},t) \quad . \tag{A1.14}$$

Since, for our study, we are interested in short time intervals compared to the time interval required for significant changes in the delayed precursor concentrations, we may write

$$\frac{\partial\, C_i(\underline{r},t)}{\partial t} \simeq 0 \qquad \text{(A1.15)}$$

and hence

$$\lambda_i\, C_i(\underline{r},t) \simeq \sum_{j=1}^{G} \beta_{ij}(\underline{r},t)\, \nu_j(\underline{r},t)\, v_j\, \Sigma_{fj}(\underline{r},t)\, n_j(\underline{r},t) \quad . \qquad \text{(A1.16)}$$

Equation (A1.13) now becomes

$$\frac{\partial\, n_\ell(\underline{r},t)}{\partial t} = \sum_{j=1}^{G} v_j\, n_j(\underline{r},t)\{\nu_j(\underline{r},t)\, \chi_{j\ell}(\underline{r},t)\, \Sigma_{fj}(\underline{r},t)$$

$$+ \Sigma_{sj}(\underline{r},t)\, f_s(\underline{r};E_j \to E_\ell;t)\} + \nabla \cdot D_\ell(\underline{r},t)\, \nabla[n_\ell(\underline{r},t)\, v_\ell]$$

$$- [\Sigma_{a\ell}(\underline{r},t) + \Sigma_{s\ell}(\underline{r},t)]\, v_\ell\, n_\ell(\underline{r},t) \quad , \qquad \text{(A1.17)}$$

where

$$\chi_{j\ell}(\underline{r},t) \equiv [1 - \beta_j(\underline{r},t)]\, \chi_{p\ell}(\underline{r},t) + \sum_{i=1}^{N} \beta_{ij}(\underline{r},t)\, \chi_{i\ell} \quad . \qquad \text{(A1.18)}$$

Equation (A1.17) models the population density of neutrons in nuclear reactors. A typical energy spectrum, as given by figure A1.1, serves to guide the choice of energy groupings in the multigroup equations. The fission process gives a fission spectrum in the high energy range. A 1/E portion results from the moderation process and is perturbed by resonance absorption. Finally, a thermal (Maxwellian distribution) spectrum results from the attainment of equilibrium of the neutrons with the reactor media.

For practical computations a much used simplification of the multigroup equations is the two-group diffusion approximation with the

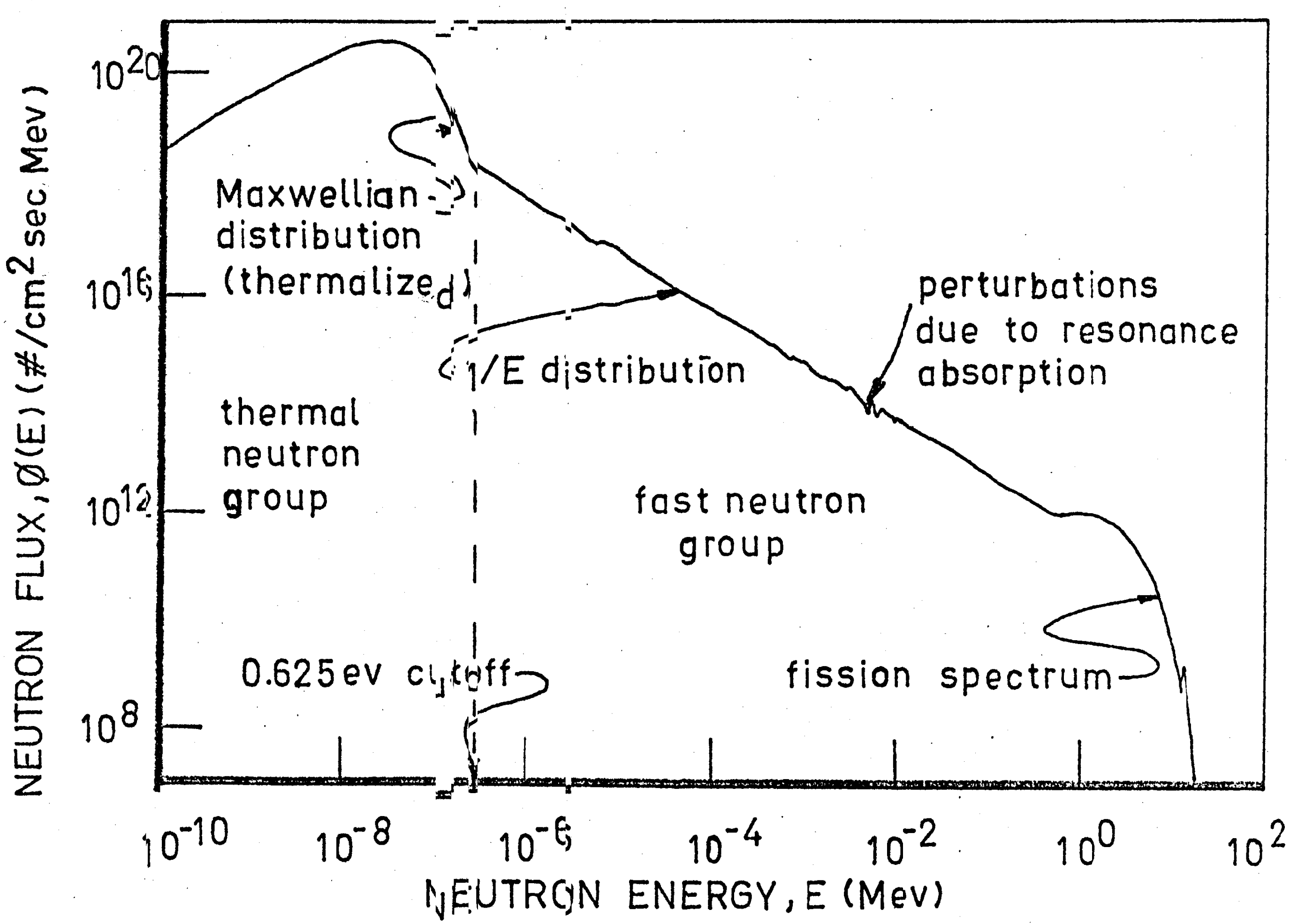

FIGURE A1.1: Typical energy spectrum for a nuclear reactor.

cutoff between groups chosen as 0.625 ev, separating the thermal portion from the epithermal and fast portions. This particular cutoff energy is experimentally convenient since, by using cadmium as a neutron filter, the relative intensities of the two neutron groups are easily measured.

A1.3 THE TWO-GROUP DIFFUSION APPROXIMATION

For the two-group diffusion approximation, ($\ell = 1,2$), we define:

$\ell = 1 \equiv$ fast group

and

$\ell = 2 \equiv$ thermal group.

The following assumptions are made:

(1) no fast fissioning: $\Sigma_{f1} = 0$;

(2) no upscatter: $f_s(E_2 \to E_1) = 0$;

(3) all fission neutrons are born in group 1: $\chi_{22} = 0$.

For convenience we drop the explicit notation of the space and time dependence and employ the above assumptions to find

$$\frac{\partial n_1}{\partial t} = n_1 v_1 \Sigma_{s1} f_s(E_1 \to E_1) + n_2 v_2 \nu_2 \Sigma_{f2} \chi_{21} + \nabla \cdot D_1 \nabla n_1 v_1 - (\Sigma_{a1} + \Sigma_{s1}) n_1 v_1$$

$$= \nabla \cdot D_1 \nabla n_1 v_1 + n_1 v_1 (\Sigma_{s1} f_s(E_1 \to E_1) - \Sigma_{s1} - \Sigma_{a1})$$

$$+ n_2 v_2 \Sigma_{f2} \chi_{21} \nu_2 \qquad \text{(A1.19)}$$

and

$$\frac{\partial n_2}{\partial t} = n_1 v_1 \Sigma_{s1} f_s(E_1 \rightarrow E_2) + n_2\, v_2\, \Sigma_{s2}\, f_s(E_2 \rightarrow E_2) + \nabla \cdot D_2 \nabla n_2 v_2$$

$$- (\Sigma_{a2} + \Sigma_{s2})\, n_2\, v_2 = n_1 v_1 \Sigma_{s1} f_s(E_1 \rightarrow E_2) - n_2\, v_2\, \Sigma_{a2}$$

$$+ \nabla \cdot D_2 \nabla n_2 v_2 \quad . \qquad \text{(A1.20)}$$

Equating coefficients with the traditional two-group equations as given ~~in chapter 4 yields: (for the fuel-coolant region)~~

$$\Sigma_{1FC} = \Sigma_{a1} + \Sigma_{s1}\, [1 - f_s(E_1 \rightarrow E_1)] = \Sigma_{a1} + \Sigma_{s1}\, f_s(E_1 \rightarrow E_2) \quad ,$$

$$\varepsilon \eta f\, \Sigma_{2FC} = \nu_2\, \chi_{21}\, \Sigma_{f2} \quad ,$$

$$\Sigma_{2FC} = \Sigma_{a2} \quad ,$$

$$p\Sigma_{1FC} = \Sigma_{s1}\, f_s(E_1 \rightarrow E_2) \quad ,$$

(for the moderator region)

$$\Sigma_{1m} = \Sigma_{a1} + \Sigma_{s1}\, f_s(E_1 \rightarrow E_2) = \Sigma_{s1}\, f_s(E_1 \rightarrow E_2) \text{ for } D_2O \, ,$$

$$\Sigma_{2m} = \Sigma_{a2} \quad .$$

Thus, given

$\nu(\underline{r},E,t)$,

$\Sigma_f(\underline{r},E,t)$,

$\Sigma_s(\underline{r},E,t)$,

$f_s(r;E' \rightarrow E;t)$,

$\chi_p(\underline{r},E,t)$,

$\chi_i(E)$,

$\beta_i(\underline{r},E,t)$,

$n(\underline{r},E,t)$

and

$v(E)$,

we can determine the two-group parameters as well as any other traditional grouping.

However, a problem arises when we try to determine the group parameters for use in determining the neutron flux distributions for a given case. We need to know the flux distribution in order to calculate the group parameters. In practice, an iterative procedure is used to give a consistent set of neutron flux distribution and group parameters. An alternate scheme is to determine the parameters experimentally; this technique has proved feasible in the past only in limited applications. The general present day practice is to theoretically determine the parameters on an iterative basis guided by what experiments can be done. The present state of the art gives errors of up to 10% in these parameters.