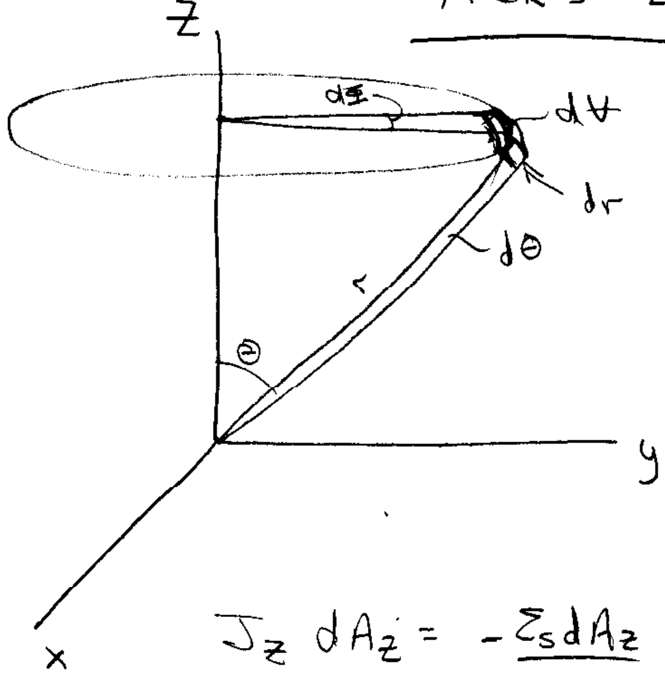
FICK'S LAW



$$dV = (r \sin\theta \, d\Phi) \cdot r\,d\theta \cdot dr$$

$$= r^2 \sin\theta \, dr \, d\theta \, d\Phi$$

$$J_z \, dA_z = -\frac{\Sigma_s \, dA_z}{4\pi} \int_{\Phi=0}^{2\pi} \int_{\theta=-\pi/2}^{\pi/2} \int_{r=0}^{\infty} e^{-\Sigma_t r} \phi(\bar{r}) (\cos\theta)(\sin\theta) \, dr \, d\theta \, d\Phi$$

$$\phi(\bar{r}) = \phi_0 + x \frac{\partial \phi}{\partial x}\Big|_0 + y \frac{\partial \phi}{\partial y}\Big|_0 + z \frac{\partial \phi}{\partial z}\Big|_0$$

$$x = r \sin\theta \cos\Phi, \quad y = r \sin\theta \sin\Phi, \quad z = r \cos\theta$$

$$\therefore \; J_z dA_z = -\frac{\Sigma_s \, dA_z}{4\pi} \iiint e^{-\Sigma_t r} \Big[\underset{(1)}{\phi_0} + \underset{(2)}{r \sin\theta \cos\Phi \frac{\partial \phi}{\partial x}\Big|_0} + \underset{(3)}{r \sin\theta \sin\Phi \frac{\partial \phi}{\partial y}\Big|_0}$$

$$+ \underset{(4)}{r \cos\theta \frac{\partial \phi}{\partial z}\Big|_0} \Big] \cos\theta \sin\theta \, dr \, d\theta \, d\Phi$$

1 $$\int_{\Phi=0}^{2\pi}\int_{\Theta=-\pi}^{0}\int_{r=0}^{\infty} e^{-\Sigma_t r}\phi_0 \cos\Theta \sin\Theta \, dr\, d\Theta\, d\Phi$$

$$-\int \cos\Theta \, d(\cos\Theta)$$

$$\Rightarrow -\int_{x=\cos(-\pi/2)}^{x=\cos(\pi/2)} x\,dx = -\frac{x^2}{2} = 0$$

3 $$\int\int\int r e^{-\Sigma_t r} \sin^2\Theta \cos\Theta \sin\Phi \, dr = \left(\int_{r=0}^{\infty} r e^{-\Sigma_t r} dr\right)\left(\ ? \ \right)$$

$$\left(\cos\Phi \Big|_0^{2\pi}\right)\frac{\partial\phi}{\partial y}\Big|_0 \Rightarrow 0$$

~ for 2

$$= 0$$

4 $$\left(\int_0^{\infty} r e^{-\Sigma_t r} dr\right)\left(\int_0^{2\pi} d\Phi\right)\left(\int_{-\pi}^{0} \cos^2\Theta \sin\Theta \, d\Theta\right)\frac{\partial\phi}{\partial z}\Big|_0$$

$$\int_0^{2\pi} d\Phi \Rightarrow 2\pi$$

$$\Rightarrow -\cos^2\Theta \, d(\cos\Theta)$$

$$\| \quad \frac{e^{-\Sigma_t r}}{\Sigma_t^2}\left(-\Sigma_t r - 1\right)\Big|_0^{\infty} = +\frac{1}{\Sigma_t^2}$$

$$= +\frac{\cos^3\Theta}{3}\Big|_{-\pi}^{0} = \frac{1}{3} - \left(-\frac{1}{3}\right) = \frac{2}{3}$$

$$\therefore \ J_z = -\frac{\Sigma_s}{4\pi} \cdot \frac{1}{\Sigma_t^2} \cdot 2\pi \cdot \frac{2}{3}\frac{\partial\phi}{\partial z}\Big|_0 = -\frac{\Sigma_s}{3\Sigma_t^2}\frac{\partial\phi}{\partial z}\Big|_0$$