ENGINEERING PHYSICS 4D3/6D3

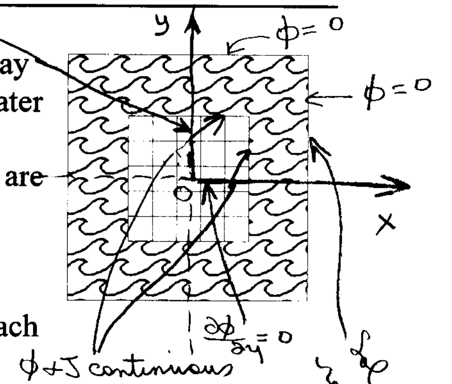
DAY CLASS — Dr. Wm. Garland

DURATION: 20 minutes

McMASTER UNIVERSITY QUIZ #2 — October 27, 1999

Special Instructions: Closed Book. All calculators and up to 8 single sided 8 ½" by 11" crib sheets are permitted.

THIS EXAMINATION PAPER INCLUDES 1 PAGE AND 1 QUESTION.



1. [14 marks total, distributed as indicated below] Consider an array of 25 identical, homogeneous fuel cells (containing U^{235}) in a water tank. The tank is square and has dimensions 100 cm x 100 cm. The fuel cylinders each have a cross section 10 cm x 10 cm and are arranged in a 5 x 5 grid in the centre of the tank as shown. The spacing between the fuel assemblies is 0 cm. Consider this a 2 dimensional case. **Assume symmetry about the centerlines.**
   a. [1 mark] Do you have to write a separate equation for each fuel cell? Why or why not?
   b. [2 marks] State the one-group transient neutron diffusion equations needed for an analytical solution.
   c. [1 mark] How many initial conditions are needed? Why?
   d. [1 mark] State the appropriate initial conditions.
   e. [1 marks] How many boundary conditions are needed? Why?
   f. [4 marks] State the appropriate boundary conditions
   g. [4 marks] Justify your use of these boundary conditions on physical terms.

Sol'n

a) No. The fuel cells are homogeneous and the central space is continuous.

b) $\frac{1}{v}\frac{\partial \phi^F}{\partial t} = D\frac{\partial^2\phi^F}{\partial x^2} + D\frac{\partial^2\phi^F}{\partial y^2} + (\nu\Sigma_f^F - \Sigma_a^F)\,\phi^F$ ← fuel region

$\frac{1}{v}\frac{\partial \phi^m}{\partial t} = D\frac{\partial^2\phi^m}{\partial x^2} + D\frac{\partial^2\phi^m}{\partial y^2} - \Sigma_a^m\,\phi^m$ – water (moderator) region

c) 2 I.C, one for each eqn. Because a differential implies an integration to solve ⇒ a constant of integration. ∴ need a known to determine this constant.

d) $\phi^m(x,y,0)$ = known, $\phi^F(x,y,0)$ = known.

e) 2 eqn's x 2 dimensions x 2nd order = 8. As per I.C., need one B.C. for every $\frac{\partial}{\partial ?}$ ⇒ 2 needed for $\frac{\partial^2\phi}{\partial x^2}$, etc.

f) $\phi = 0$ at boundaries, ie ① $\phi(x,50,t) = 0$, ② $\phi(50,y,t) = 0$

$\frac{\partial\phi}{\partial x} = \frac{\partial\phi}{\partial y} = 0$ at line of symmetry, ie ③ $\left.\frac{\partial\phi}{\partial x}\right|_{0,y,t}$ = ④ $\left.\frac{\partial\phi}{\partial y}\right|_{x,0,t} = 0$

$J_x = -D\frac{\partial\phi}{\partial x}$, etc

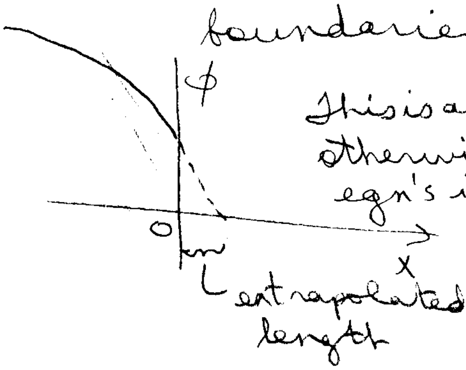
ϕ & J at interfaces continuous, ie,

⑤ $\phi^F(25,y,t) = \phi^m(25,y,t)$, ⑥ $\phi^F(x,25,t) = \phi^m(x,25,t)$

⑦ J_x^F " " = J_x^m " " , ⑧ J_y^F " = J_y^m " ⟶ more

g) <u>B.C. Justification</u>

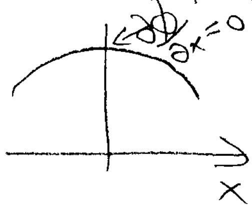
$\phi = 0$ at boundaries (actually at the extrapolated boundaries)



This is an approximation we employ since otherwise we'd need to solve the transport eqn's in the vacuum region.

ϕ, J continuous at interfaces - because there is no accumulation of neutrons at the interfaces (or sinks)

$\frac{\partial \phi}{\partial x} = 0$ at line of symmetry: Valid if there are no δ type sources at the line.



One side is a mirror image of the other ~~so~~ $\frac{\partial \phi}{\partial x} = 0$