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ENGINEERING PHYSICS 4D3/6D3

DAY CLASS — Dr. Wm. Garland

DURATION: 20 minutes

McMASTER UNIVERSITY QUIZ #2 — 2005-11-10

Special Instructions:

1. Closed Book. All calculators and up to 6 single sided 8 ½" by 11" crib sheets are permitted.
2. The value of each part is as indicated. TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 1 PAGE AND 1 MULTI-PART QUESTION. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. (Based on assignment #4, question 3) Consider a homogeneous slab of fuel / moderator coolant with known properties, thickness 'a' and infinite in the other 2 dimensions. Assume one-speed neutrons. For this solution, devise a "controller" to adjust the absorption term (not the fission term) so that a steady state is reached.
   a. Write the governing differential partial differential equation (PDE) for the transient solution.
   b. Write the finite difference form of the PDE using the semi-implicit method. Adopt an eqi-spaced grid.
   c. Outline a solution algorithm.
   d. Sketch the center-line flux and absorption as a function of time that you might expect.

Solution: (obviously, the actual programming done below is not expected for the quiz and is offered here for completeness)

a.

The governing differential equation for transient solution:

$$\frac{1}{v}\frac{\partial}{\partial t}\Phi(x,t) = D\frac{\partial^2\Phi(x,t)}{\partial x^2} - \Sigma_a\Phi(x,t) + v\Sigma_f\Phi(x,t)$$

b.

For equal space grid:

$$\frac{1}{v}\frac{\Phi(i)^{t+\Delta t} - \Phi(i)^t}{\Delta t} = D\frac{\Phi(i+1)^t - 2\Phi(i)^{t+\Delta t} + \Phi(i-1)^{t+\Delta t}}{\Delta x^2} - \Sigma_a\Phi(i)^{t+\Delta t} + v\Sigma_f\Phi(i)^t$$

The flux value for node (i-1) has been updated when calculating node i. As such, the updated value will be used for this node. This method is called Gauss-Seidel method, also called semi-implicit method.

c.

$$S(i) = v\Sigma_f\Phi(i)^t$$

$$\Phi(i)^{t+\Delta t} = \frac{\Phi(i)^t + v\Delta t(\frac{D}{\Delta x^2}\Phi(i+1)^t + \frac{D}{\Delta x^2}\Phi(i-1)^{t+\Delta t} + S(i))}{1 + v\Delta t(\frac{D}{\Delta x^2} + \Sigma_a)}$$

To solve this transient equation, let's devise a "controller" to adjust the absorption term:

$$\Sigma_a^{t=\Delta t} = \Sigma_a^t + a(\Phi(s)^{t+\Delta t} - \Phi(s)_{set}) + b\frac{\Phi(s)^{t+\Delta t} - \Phi(s)^t}{\Delta t}$$

$\Phi(s)^{t+\Delta t}$ is the flux value at the specified point, for example, the centre of the slab.

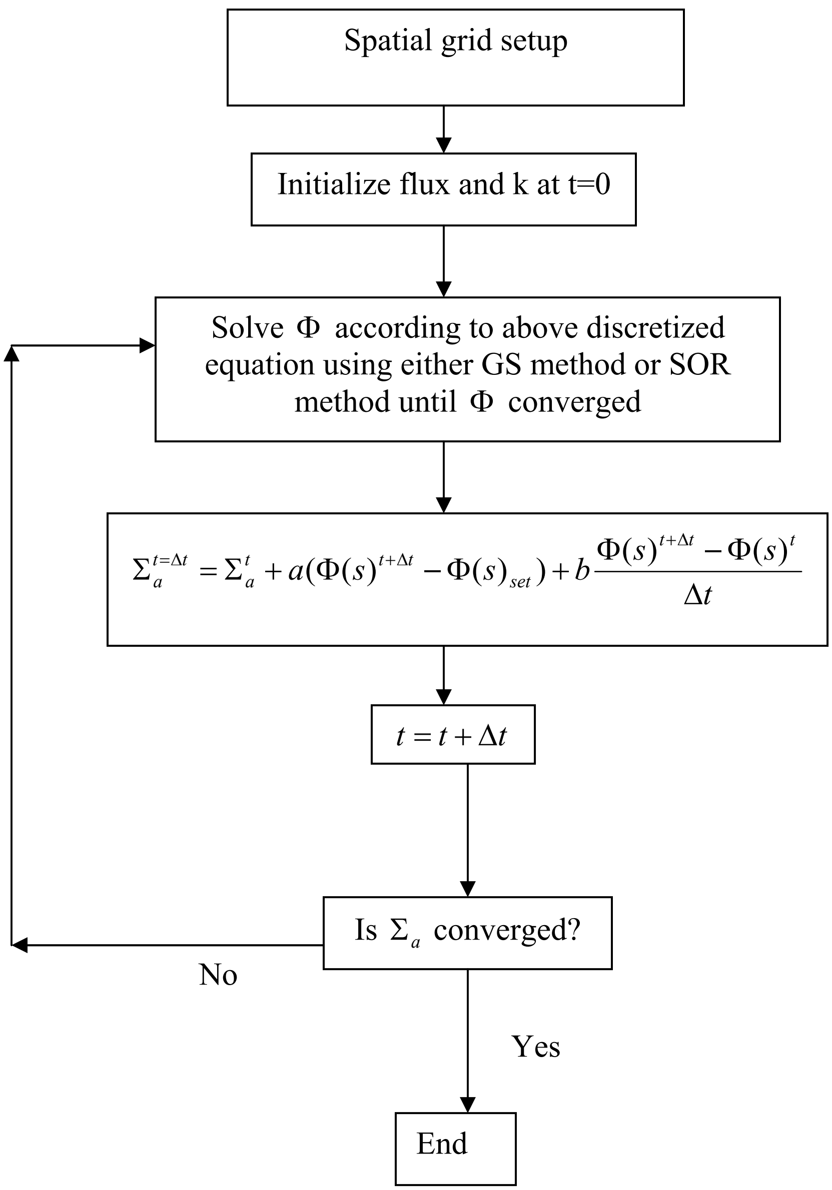
$\Phi(s)_{set}$ is the set value of the flux of aforementioned point, for example, 1.

a, b are constants. We can set a=0.001, b=0.0001.

Convergence criteria:

$$\left|\frac{\Sigma_a^{t+\Delta t} - \Sigma_a^t}{\Sigma_a^t}\right| \leq \varepsilon$$

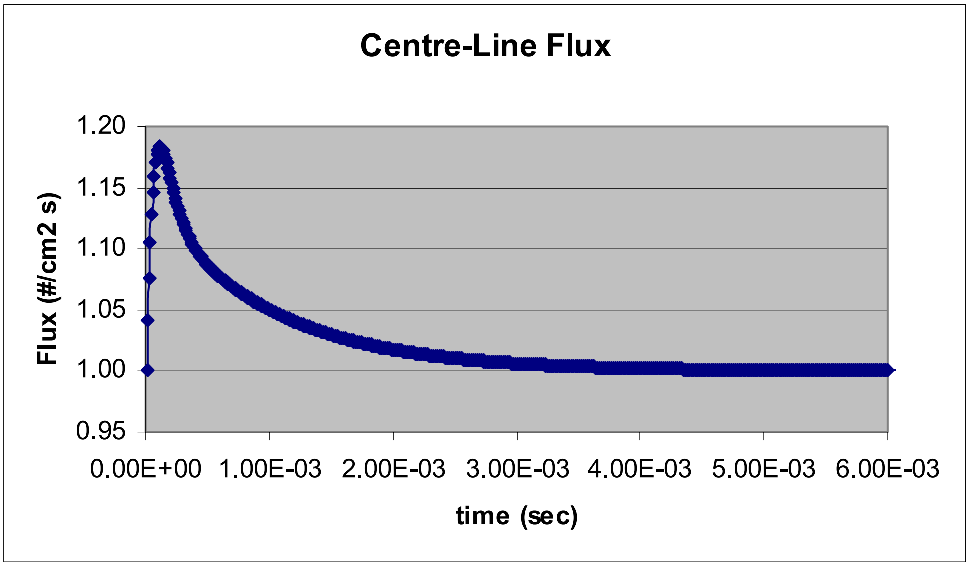
The reactivity of reactor is determined by fission, absorption and leakage. When we choose to devise the controller to adjust the absorption, we are actually control the reactivity control rods, such as adjusters and absorbers, or a liquid zone controller in a CANDU reactor.

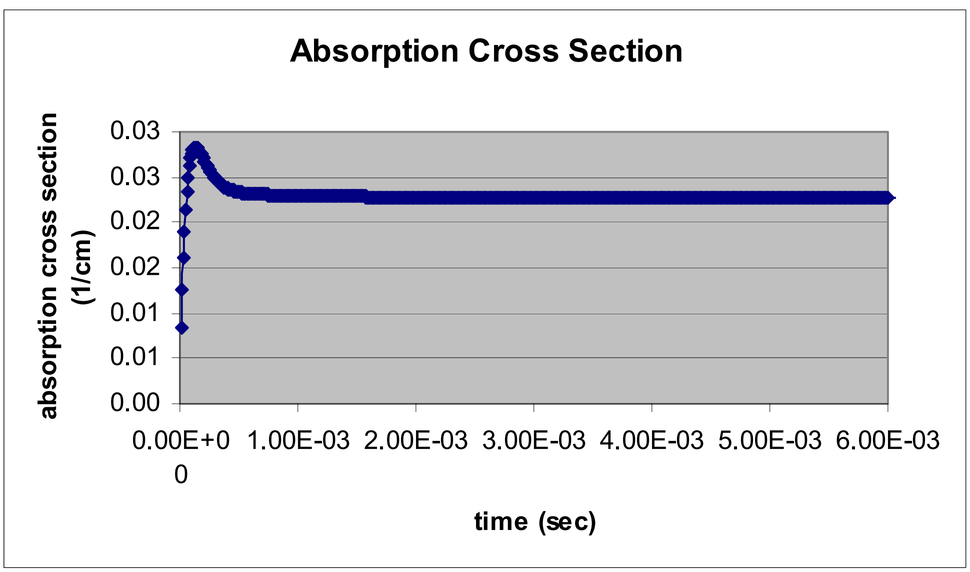


d.

The expected centre-line flux and absorption as a function of time would be an adjustment before converge.

As I'm taking advantage of sending the Quiz answer by mail, I coded the above algorithm and found the profiles for centre-line flux and the absorption cross section.





It has been noted that the value of a, b and ε should be adjust properly to get a stable solution. The values used are:

a=0.001

b=1e-6

ε=1e-4

For your information the source code is also attached.

```
c       Quiz #2 Tony Liang
        program quiz2
        dimension s(100),flux(100),fluxnew(100)
        open(1,file='sa.dat',status='new')
        open(2,file='flux.dat',status='new')
        N=10
        x=120
        Aold=0.7
        v0=2.98
        sf=0.00395*1.85
        sa=0.00395*2.11
        str=0.00395*6.8
        D=1/(3*str)
        v=220000
        t=0
        deltaT=1e-5
        delta=x/N
        maxflux=1.0
        fluxnew(1)=0
        fluxnew(11)=0
        flux(1)=0
        flux(11)=0
        a=0.001
        b=1e-6
        fluxset=1.0

        do i=2,10
        flux(i)=1.0
        enddo

100     t=t+deltaT
        write(1,*) t,sa
        write(2,*) t,flux(6)
        fluxsumold=0
        do i=2,10
        s(i)=1/Aold*v0*sf*flux(i)
        fluxsumold=fluxsumold+flux(i)
        enddo
        sumfluxnew=0
        do i=2,10
        fluxnew(i)=(flux(i)+v*deltaT*(D/delta/delta*fluxnew(i-1)+
     + D/delta/delta*flux(i+1)+s(i)))/(1+v*deltaT*(2*D/delta/delta+sa))
        sumfluxnew=sumfluxnew+fluxnew(i)
        enddo

        sanew=sa+a*(fluxnew(6)-fluxset)+b*(fluxnew(6)-flux(6))/deltaT
        if (abs((sanew-sa)/sa).ge.1e-4
     +  .or.abs(fluxnew(6)-fluxset).gt.1e-4) then
         do i=2,10
          flux(i)=fluxnew(i)
         enddo
         sa=sanew
        goto 100
        else
        endif

        stop
        end
```