ENGINEERING PHYSICS 4D3/6D3

DAY CLASS — Dr. Wm. Garland
DURATION: 50 minutes
McMASTER UNIVERSITY MIDTERM EXAMINATION — November 5, 1998

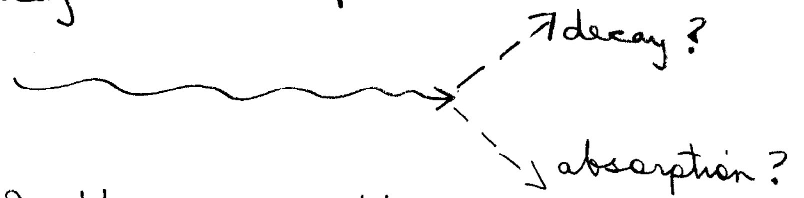
Special Instructions:

1. Closed Book. All calculators and up to 6 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions. Place your answers on the exam sheets; use additional pages if necessary.
3. The value of each question is as indicated. TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 4 PAGES AND 3 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [30 marks] A free neutron beta decays with a half-life of 11.7 minutes. Determine the relative probability that a thermal neutron will undergo beta decay before being absorbed in an infinite medium. Calculate this probability using water, $\Sigma_a = 0.022\text{cm}^{-1}$, as the medium. HINT: Distance = speed x time. Compare the following probabilities:
   - the probability of travelling some distance, x, without being absorbed or decaying and then decaying at x
   - the probability of travelling distance, x, without being absorbed or decaying and then getting absorbed at x.

$$dn = \underbrace{-\lambda n dt}_{\text{decay}} - \underbrace{\Sigma_a n dx}_{\text{absorption}} \quad , \quad dx = v dt$$



$$\therefore dn = -\lambda n dt - \Sigma_a n v dt = -(\lambda + \Sigma_a v) dt$$

$$\therefore n = n(0) e^{-(\lambda + \Sigma_a v)t}$$

ratio of decay/absorption $= \dfrac{\lambda \cancel{n} \cancel{dt}}{v \Sigma_a \cancel{n} \cancel{dt}} = \dfrac{\lambda}{v \Sigma_a}$

$v = 2.2 \times 10^5$ cm/sec.

$\Sigma_a = 0.022$ cm^{-1}

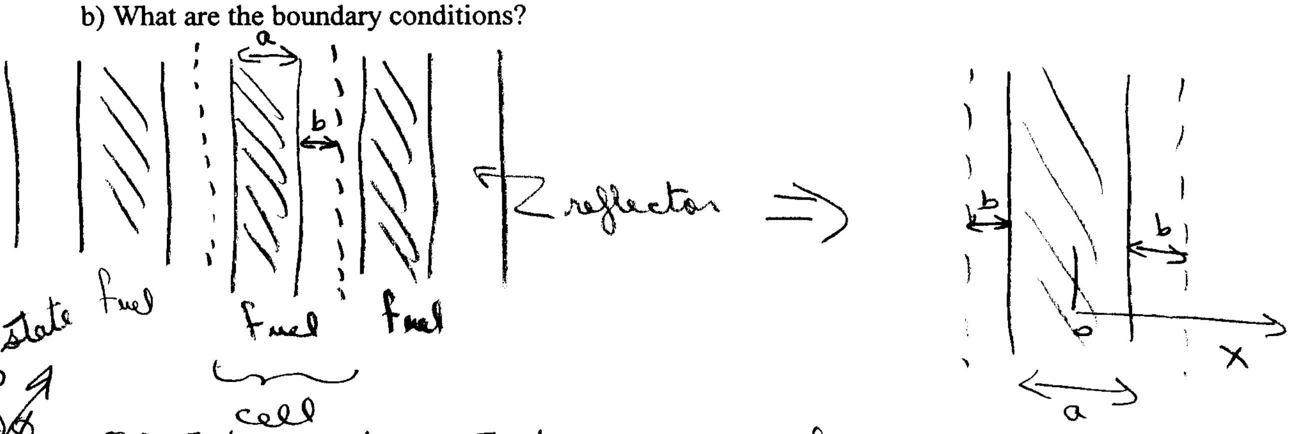
$\lambda = \dfrac{\ln 2}{T_{\frac{1}{2}}} = \dfrac{0.693}{11.7 \times 60}$ sec^{-1} $= 9.9 \times 10^{-4}$ sec^{-1}

$\therefore \dfrac{\lambda}{v\Sigma_a} = 2.04 \times 10^{-7}$, ie decay is not likely.

2. [20 marks] Consider a large one dimensional reactor composed of many replicated identical cells, each containing fuel and moderator. Each cell consists of a central fuel region of thickness "a" surrounded on either side by a reflector of thickness "b". Near the centre of the reactor, we can assume that one cell looks and behaves like its neighbours since the reactor is large. Thus, the flux distribution in each central cell can be calculated independently.

a) What are the governing flux equations for the steady state?

b) What are the boundary conditions?



steady state

$$\frac{1}{v}\cancel{\frac{\partial \phi}{\partial t}} = \nabla\cdot D\nabla\phi - \Sigma_a\phi + \nu\Sigma_f\phi \quad \leftarrow \text{general}$$

(a) fuel region: $$D_F\frac{\partial^2\phi_F}{\partial x^2} + (\nu\Sigma_{f_F} - \Sigma_{a_F})\,\phi_F = 0$$

reflector region: $$D_R\frac{\partial^2\phi_R}{\partial x^2} - \Sigma_{a_R}\phi_R = 0$$

(b) Boundary Conditions:

(1) $\left.\frac{\partial\phi_F}{\partial x}\right|_{x=0} = 0$ (symmetry ∴ slope of ϕ at $x=0$ is 0.)

continuity of flux + current at the interface

(2) $\left.\phi_F\right|_{x=a/2} = \left.\phi_R\right|_{x=a/2}$

(3) $\left.J_F\right|_{x=a/2} = \left.J_R\right|_{x=a/2} \Rightarrow D_F\left.\frac{\partial\phi_F}{\partial x}\right|_{x=a/2} \simeq D_R\left.\frac{\partial\phi_R}{\partial x}\right|_{x=a/2}$

(4) Reflection at the cell boundary (ie since one cell look like another, neutrons leaving the cell must have a counterpart entering from the neighbour cell. Hence $\left.J_R\right|_{x=a/2+b} = 0 \Rightarrow \left.\frac{\partial\phi_R}{\partial x}\right|_{x=a/2+b} = 0$.

3. [50 marks] Outline a computer program to solve the one group neutron space-time diffusion equation in an infinite slab reactor (extrapolated thickness "a") with heterogeneous properties, ie space-dependent parameters. Consider control, poisons and fuel depletion aspects. Focus on:

a) the governing equations for:
    neutrons (5 marks)
    xenon and iodine (5 marks)
    fuel depletion (5 marks)
    a simple control mechanism to take the reactor from a low initial flux to full power (5 marks)
b) the boundary conditions (5 marks)
c) the initial conditions (5 marks)
d) the finite difference scheme (5 marks)
e) the solution algorithm (including a flow chart) showing the overall solution and how the equations interact (15 marks).

Don't get hung up on details.

(a) $\frac{1}{v}\frac{\partial\phi}{\partial t} = \nabla\cdot D\nabla\phi + (\nu\Sigma_f - \Sigma_a)\phi$ (neutron flux)

(all coefficients (D, Σ) are space dependent and, over the long term may be time dependent)

$\frac{\partial I}{\partial t} = \gamma_I \Sigma_f \phi - \lambda_I I$ (iodine), $\frac{\partial X}{\partial t} = \gamma_X \Sigma_f \phi + \lambda_I I - \lambda_X X - \sigma_a \phi X$ (xenon)

$\frac{\partial N_f}{\partial t} = - N_f \sigma_a^f \phi$ (fuel)

Control: Let's put a control rod at some point in space in the reactor (the control point). Thus, Σ_a is a function of control rod insertion (Z):

$\Sigma_a = \Sigma_{a0} + \Sigma_a^c\,(Z/Z_0)$ where Z_0 = fully inserted.

$$+\ Z = \underset{\substack{\uparrow \\ \text{Say } 1/2 \text{ inserted}}}{Z_{\text{initial}}} + \overbrace{\underset{\substack{\uparrow \\ \text{a constant}}}{\text{gain}} \times (\phi_\xi - \underset{\substack{\uparrow \\ \text{Say } 100\% \text{ full power}}}{\phi_{\text{setpoint}}})}^{\Delta Z}$$

So if $\phi_\xi > \phi_{\text{setpoint}}$, ΔZ is +ve, ie $Z \uparrow$, ie insertion increases.

($Z = 0$ if control rod is fully withdrawn,
$Z = Z_0$ " " " " inserted).

Start the calculation at a low power + the above controller will home in on the setpoint flux.

b) B.C. $\phi = 0$ at extrapolated length ($\pm a/2$)
No B.C. needed for the other equations

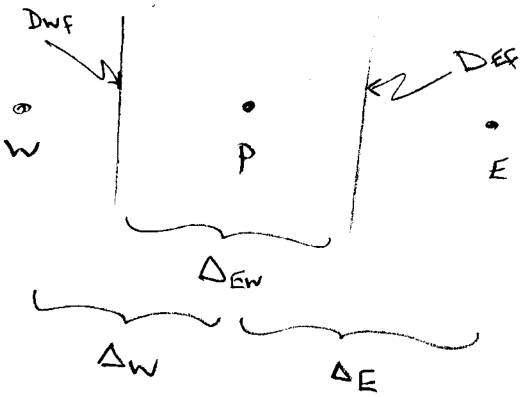
c) I.C. $\phi(x, \overset{t}{0}) = f_n(x)$, say cosine with some amplitude (say, low power).

$X(x,0) = I(x,0) = 0$ (no poisons at $t=0$)

$N_F(x,0) = N_{F_0}$ (fresh fuel).

d)
$$\frac{1}{v}\frac{\phi_P^{t+\Delta t} - \phi_P^t}{\Delta t} = \frac{D_{EF}\,\phi_E}{\Delta_E\,\Delta_{EW}} - \left(\frac{D_{EF}}{\Delta_E} + \frac{D_{WF}}{\Delta_W}\right)\frac{\phi_P}{\Delta_{EW}} + \frac{D_{WF}}{\Delta_W}\frac{\phi_W}{\Delta_{EW}} + \left[(\nu\Sigma_f)_P - \Sigma_{a_P}\right]\phi_P$$

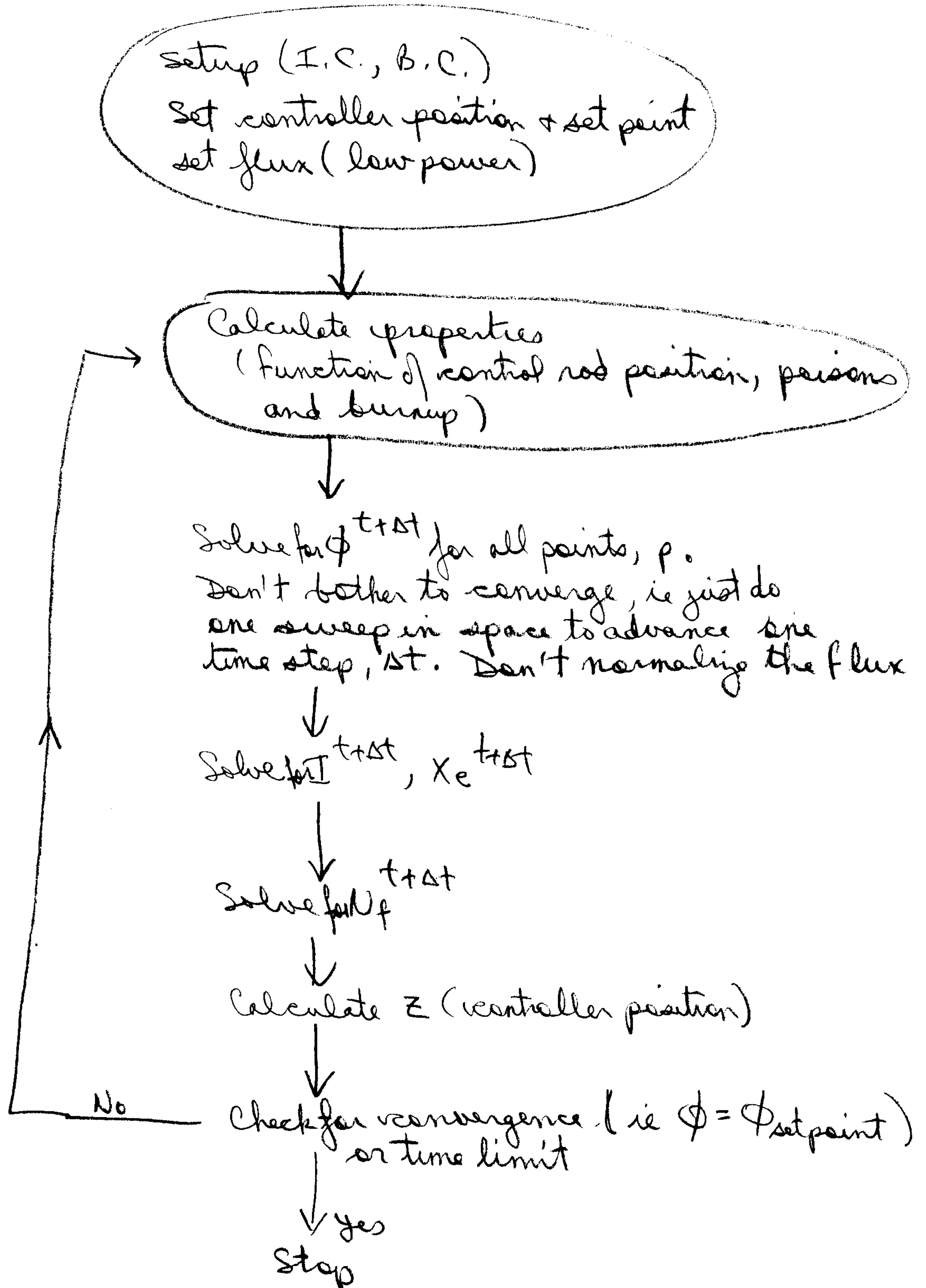
where $D_{WF} = \frac{1}{2}(D_E + D_P)$, $D_{EF} = \frac{1}{2}(D_W + D_P)$



Fuel regions only
$$\left\{\begin{aligned}
\frac{I_P^{t+\Delta t} - I_P^{t}}{\Delta t} &= \gamma_I \Sigma_{f_P}\phi_P - \lambda_I I_P \\
\frac{X_P^{t+\Delta t} - X_P^{t}}{\Delta t} &= \gamma_X \Sigma_{f_P}\phi_P + \lambda_I I_P - \lambda_X X_P - \sigma_{a_P}\phi_P X_P \\
\frac{N_{F_P}^{t+\Delta t} - N_{f_P}^{t}}{\Delta t} &= -N_{FP}\sigma_{a_P}^{F}\phi_P
\end{aligned}\right.$$

continued →

(e)



You can save on computational time by noting that N_F is changing very slowly compared to I + Xe so the N_F update can be done infrequently (say every hour). Similarly I + Xe are changing slowly w.r.t. ϕ so do it only every minute or so. ϕ is changing rapidly so track it carefully to full power. What at full power, you can switch to a steady state version of the flux calculation.

— end —