MIDTERM TEST
ENGINEERING PHYSICS 4D3

Oct. 20/88

Nuclear Reactor Systems Analysis
Lecturer: Wm. J. Garland

Name: ____________________

I.D.: ____________________

Duration: 50 minutes

Special Instructions:

1. Do all 5 questions.
2. Breakdown of marks indicated to the right of the question.
3. Closed book; crib sheets and calculators permitted.

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1. a) Why are neutrons so important in nuclear reactors? (3)

neutrons are the catalyst for the chain reaction.

b) Fill in the attached neutron cycle chart. (10)

c) Describe part (b) in words. (7)

Need to control neutrons to control reaction + hence power. Thus need to understand + model neutron population.

2. a) Explain the meaning of (term by term plus total concept): (15)

$$\frac{dn(t)}{dt} = \frac{1}{v}\frac{d\phi_T(t)}{dt} = \left((1-\beta)k_\infty - 1\right)\Sigma_a\phi_T(t) + p\sum_i \lambda_i C_i(t)$$

8

and

$$\frac{dC_i(t)}{dt} = \beta_i\frac{k_\infty}{p}\Sigma_a\phi_T(t) - \lambda_i C_i(t)$$

7

b) Describe in the form of a reaction equation: (5)

Indium (In), having 49 protons and 67 neutrons, decays to tin (Sn) which has 50 protons and 6~~7~~6 neutrons, with the release of an electron, a neutrino and some gamma energy.

$${}_{49}In^{116} \xrightarrow{\beta^-} {}_{50}Sn^{116} + \nu_0 + \gamma + e^-$$

3. Based on mathematical-physical reasoning, describe how you would specify the <u>limits on reactivity insertion</u> to ensure that prompt critical cannot occur in a fission reactor. Justify your reasoning. (20)

pt. kinetics eqn is $\frac{dn}{dt} = \left(\frac{\rho-\beta}{\Lambda}\right) n(t) + \sum_{i=1}^{6} \lambda_i C_i$

when $\rho > \beta$, then $n(t) \sim e^{\frac{(\rho-\beta)t}{\Lambda}}$ ← exponential growth ie more than critical on prompt neutrons alone. thus limit +ve $\Delta\rho$ to be $< \beta$.

4. A bare, homogeneous cylindrical reactor can be characterized by: (20)

D = 10 cm., Σ_a = 0.15 cm^{-1}, H = R with a 10% neutron leakage probability. What are its' critical dimensions?

$\frac{1}{v}\frac{\partial \phi}{\partial t} = -\Sigma_a\phi + \nu\Sigma_f\phi + D\nabla^2\phi$, $\nabla^2\phi + B_g^2\phi = 0$

For bare cylinder: $B_g^2 = \left(\frac{\pi}{H}\right)^2 + \left(\frac{\nu_0}{R}\right)^2$

5. <u>Outline</u> a computer program to solve the one speed neutron space-time diffusion equation in a finite cylindrical reactor with a heterogeneous core/coolant/moderator, i.e. space-time dependent parameters. Focus on:

a) the governing equations (4)

b) the boundary conditions (4)

c) the initial conditions (4)

d) the finite difference scheme (4)

e) the solution algorithm (including a flow chart) (4)

(See reverse) →

Include the delayed precursor equations . Ignore thermalhydraulic effects but consider the possible effects of reactor control. Remember this is a space-time problem. Don't get hung up on details!

$$0 = \underset{\uparrow}{(\nu\Sigma_f - \Sigma_a)}\phi + \underset{-DB^2\phi}{D\nabla^2\phi} \Rightarrow \frac{\nu\Sigma_f}{\Sigma_a} - 1 + \underset{L^2}{\left(\frac{D}{\Sigma_a}\right)} B^2 = 0$$

Fission absorption leakage

$$\frac{\frac{\nu\Sigma_f}{\Sigma_a}}{(1 + L^2B^2)} = 1 \Rightarrow \frac{1}{1+L^2B^2}$$ is the prob. of non leakage $= 0.9$

$\therefore \frac{1}{.9} = 1 + L^2B^2 \quad \therefore B^2 = \frac{1/.9 - 1}{\left(\frac{10}{0.15}\right)}$

$= .001667$

$= \frac{(\pi)^2 + (\nu_0)^2}{H^2}$ ←

THE END

$\therefore H = R = \sqrt{\frac{[(\pi)^2 + (\nu_0)^2]}{.001667}}$ cm.

$= 96.91$

a) $$\frac{1}{v}\frac{\partial \phi(\underline{r},t)}{\partial t} = \nabla\cdot D(\underline{r},t)\,\nabla\phi(\underline{r},t) - \Sigma_a(\underline{r},t)\,\phi(\underline{r},t) + \nu(\underline{r},t)\,(1-\beta(\underline{r},t))\,\Sigma_f(\underline{r},t)\,\phi(\underline{r},t) + \sum_{i=1}^{6}\lambda_i\,C_i(\underline{r},t)$$

$$\frac{\partial C_i(\underline{r},t)}{\partial t} = \beta_i(r,t)\,\Sigma_a(\underline{r},t)\,\phi(\underline{r},t) - \lambda_i\,C_i(\underline{r},t)$$

where $\nabla\cdot D\nabla\phi = D\nabla^2\phi + \nabla D\cdot\nabla\phi$

express in cylindrical geometry:

$$\nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2}$$

etc

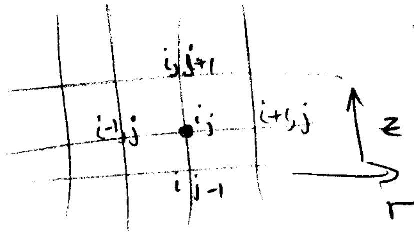
b) $\phi(\tilde{R},t) = 0$, $\phi(\pm H/2, t) = 0$

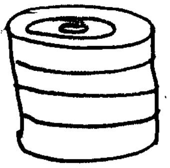
Don't really need to use continuity of flux since we are solving numerically + not trying to match solution at interface.

c) $\phi(\underline{r},0)$ = given.

$C(\underline{r},0)$ = given

d) central difference in space : $$\frac{\partial\phi}{\partial r} = \frac{\phi^t_{i+1,j} - \phi_{i-1,j}}{2\Delta r}$$





$$\frac{\partial^2\phi}{\partial r^2} = \frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{\Delta r^2}$$

~ for z. $$+\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial\phi}{\partial r} = \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r}$$

$$\frac{\partial\phi}{\partial t} = \frac{\phi^{t+\Delta t}_{i,j} - \phi^t_{i,j}}{\Delta t}$$

$\Downarrow$

$\frac{\partial\underline{\phi}}{\partial t} = \underline{\underline{A}}\,\underline{\phi}$ Explicit

$\underline{\phi}^{t+\Delta t} = \underline{\phi}^t + \Delta t\,\underline{\underline{A}}\,\underline{\phi}$

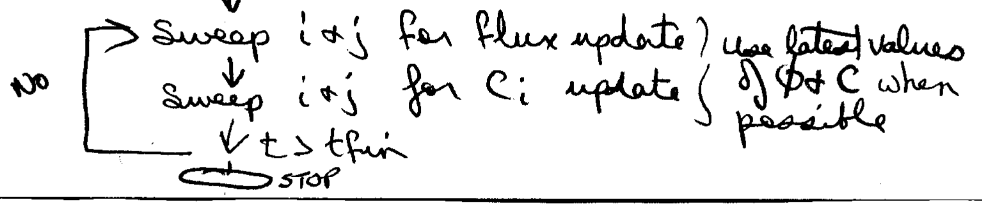
or $\underline{\phi}^{t+\Delta t} = [I - \Delta t\,\underline{\underline{A}}]^{-1}\underline{\phi}^t \leftarrow$ for C_i

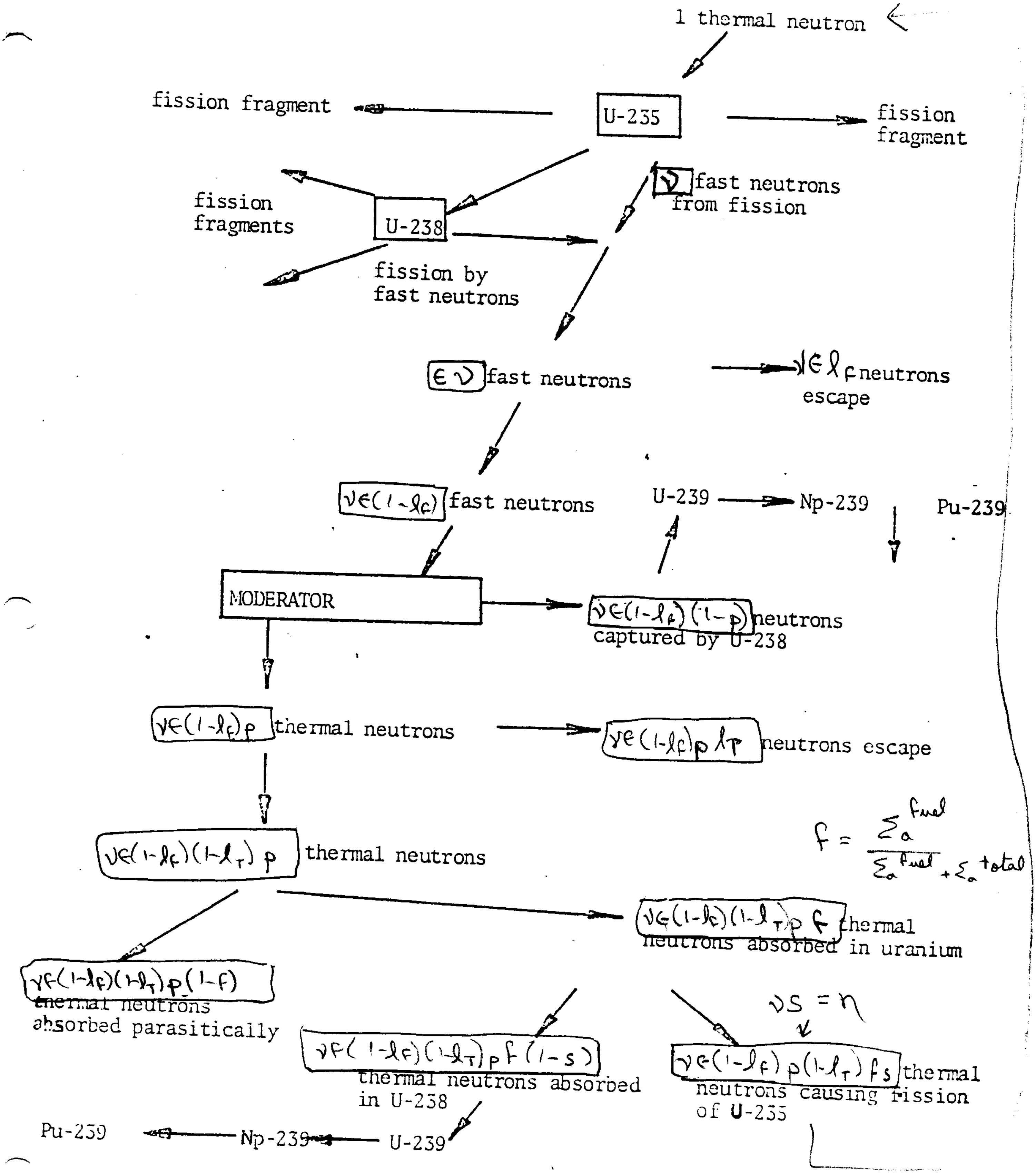
At $r=0$, evaluate $\frac{1}{r}\frac{\partial\phi}{\partial r}$ using L'Hopital's rule $= \frac{\partial^2\phi}{\partial r^2}$

Use symmetry wherever possible $\Rightarrow$ at $r=0$, $\phi_{i-1} = \phi_{i+1}$

Could use x, y, z geometry but will have edge problems.

e) flow chart:





Schematic representation of the neutron cycle in the chain reaction of uranium fission.