Solution

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ENGINEERING PHYSICS 4D3/6D3

DAY CLASS Dr. Wm. Garland

DURATION: 50 minutes

McMASTER UNIVERSITY MIDTERM EXAMINATION November 2, 2004

Special Instructions:

1. Closed Book. All calculators and up to 6 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions.
3. The value of each question is as indicated. TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 2 PAGES AND 3 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [20 marks] For a homogeneous, critical, one dimensional, bare slab reactor (modelled by the one-speed neutron diffusion equation), what is the steady state xenon spatial distribution?

$\frac{\partial X}{\partial t} = \gamma_x \Sigma_f \phi + \lambda_I I - \lambda_x X + \sigma_a^X \phi X$, $\frac{\partial I}{\partial t} = \gamma_I \Sigma_f \phi - \lambda_I I$

Where $\phi = \phi_0 \cos Bx$ as usual. $B = \pi/a$.

In steady state: $I = \frac{\gamma_I \Sigma_f \phi_0}{\lambda_I} \cos Bx$

$\therefore$ for Xe: $0 = \gamma_x \Sigma_f \phi + \gamma_I \Sigma_f \phi - \lambda_x X - \sigma_a^X \phi X$

$$\therefore \boxed{X = \frac{(\gamma_x + \gamma_I)\Sigma_f \phi_0 \cos Bx}{\lambda_x + \sigma_a^X \phi_0 \cos Bx}}$$

It is not a simple cosine.

2. [20 marks] A bare homogeneous cubic reactor can be characterized by one group neutron diffusion, D = 10 cm., $\Sigma_a = 0.1$ cm.$^{-1}$, height = width = length = 100 cm. What is the neutron non-leakage probability, P_{NL}? [Hint: $P_{NL} = 1 / (1 + B_g^2 L^2)$]

$B_g^2 = \left(\frac{\pi}{H}\right)^2 + \left(\frac{\pi}{L}\right)^2 + \left(\frac{\pi}{W}\right)^2 = 3\left(\frac{\pi}{H}\right)^2$ for a cube.

Non-leakage probability $= \frac{1}{1 + B_g^2 L^2}$, $L^2 = \frac{D}{\Sigma_a} = \frac{10}{.1} = 100\ cm^2$

$$= \frac{1}{1 + 3\left(\frac{\pi}{100}\right)^2 \cdot 100} = \frac{1}{1 + \frac{3\pi^2}{100}} = 0.772$$

3. For the one-group transient neutron diffusion model of a one dimensional, homogeneous, bare slab reactor:

   a. [20 marks] State the neutron balance equation and the appropriate initial and boundary conditions.

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\frac{\partial^2 \phi}{\partial x^2} + (\nu \Sigma_f - \Sigma_a)\phi$$

BC: $\phi(\pm a/2, t) = 0$ or $\left.\frac{\partial \phi}{\partial x}\right|_{x=0} = 0$ or some combination

IC: $\phi(x,0)$ = given

   b. [20 marks] Derive the stability criteria for the explicit numerical scheme.

$$\frac{1}{v}\frac{\phi_P^{t+\Delta t} - \phi_P^{t}}{\Delta t} = \frac{D}{\Delta x^2}\left(\phi_W^t - 2\phi_P^t + \phi_E^t\right) + (\nu\Sigma_f - \Sigma_a)\phi_P^t$$

$$\therefore \phi_P^{t+\Delta t} = \frac{v \Delta t D}{\Delta x^2}\left(\phi_W^t + \phi_E^t\right) + \underbrace{\left(1 - v\Delta t\left(\frac{2D}{\Delta x^2} - \nu\Sigma_f + \Sigma_a\right)\right)}_{\text{gives instability if } < 0}\phi_P^t$$

$$\therefore \text{ for stability: } \Delta t \le \frac{\Delta x^2}{v\left(2D - \nu\Sigma_f \Delta x^2 + \Sigma_a \Delta x^2\right)}$$

   c. [20 marks] Show how this condition is relaxed when an implicit scheme is used.

$$\phi_P^{t+\Delta t} = \frac{v\Delta t}{\Delta x^2}\left(\phi_W^{t+\Delta t} + \phi_E^{t+\Delta t}\right) + \phi_P^t + \left(-v\Delta t\left(\frac{2D}{\Delta x^2} - \nu\Sigma_f + \Sigma_a\right)\right)\phi_P^{t+\Delta t}$$

$$\therefore \phi_P^{t+\Delta t} = \frac{\frac{v\Delta t D}{\Delta x^2}\left(\phi_W^{t+\Delta t} + \phi_E^{t+\Delta t}\right) + \phi_P^t}{1 + v\Delta t\left(2D/\Delta x^2 - \nu\Sigma_f + \Sigma_a\right)}$$

This will be well behaved even for large Δt. Watch out for positive reactivity, though, where $\nu\Sigma_f$ is big.

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