Final exam 1998 - Solutions

#1

(a) $\phi \equiv n v$ (scalar) $[\equiv]$ #/cm^2 sec

$J \equiv n \underline{v}$ (vector) $[\equiv]$ #/cm^2 sec.

(b) critical is $K=1$ or $\rho=0$, ie steady state flux where births = deaths.

prompt critical is when the reactor is critical on prompt neutrons only, ie in the point kinetics eqn: $\frac{\partial n}{\partial t} = \frac{\rho-\beta}{\Lambda} n + \sum_i \lambda_i C_i$

$\rho = \beta$. Thus reactor is supercritical due to excess delayed neutrons. When $\rho \gtrsim \beta$, $\frac{\partial n}{\partial t}$ increases with a short time constant ($\sim \Lambda$).

(c) $\Sigma_s(E_1 \to E_2)$ is scattering down from group 1 to group 2.

$\Sigma_s(E_2 \to E_1)$ " " up " " 2 " 1.

(d) $\eta = \frac{\nu \Sigma_F^{fuel}}{\Sigma_a^{fuel}}$, $f = \frac{\Sigma_a^{fuel}}{\Sigma_a^{fuel + all\ else}}$

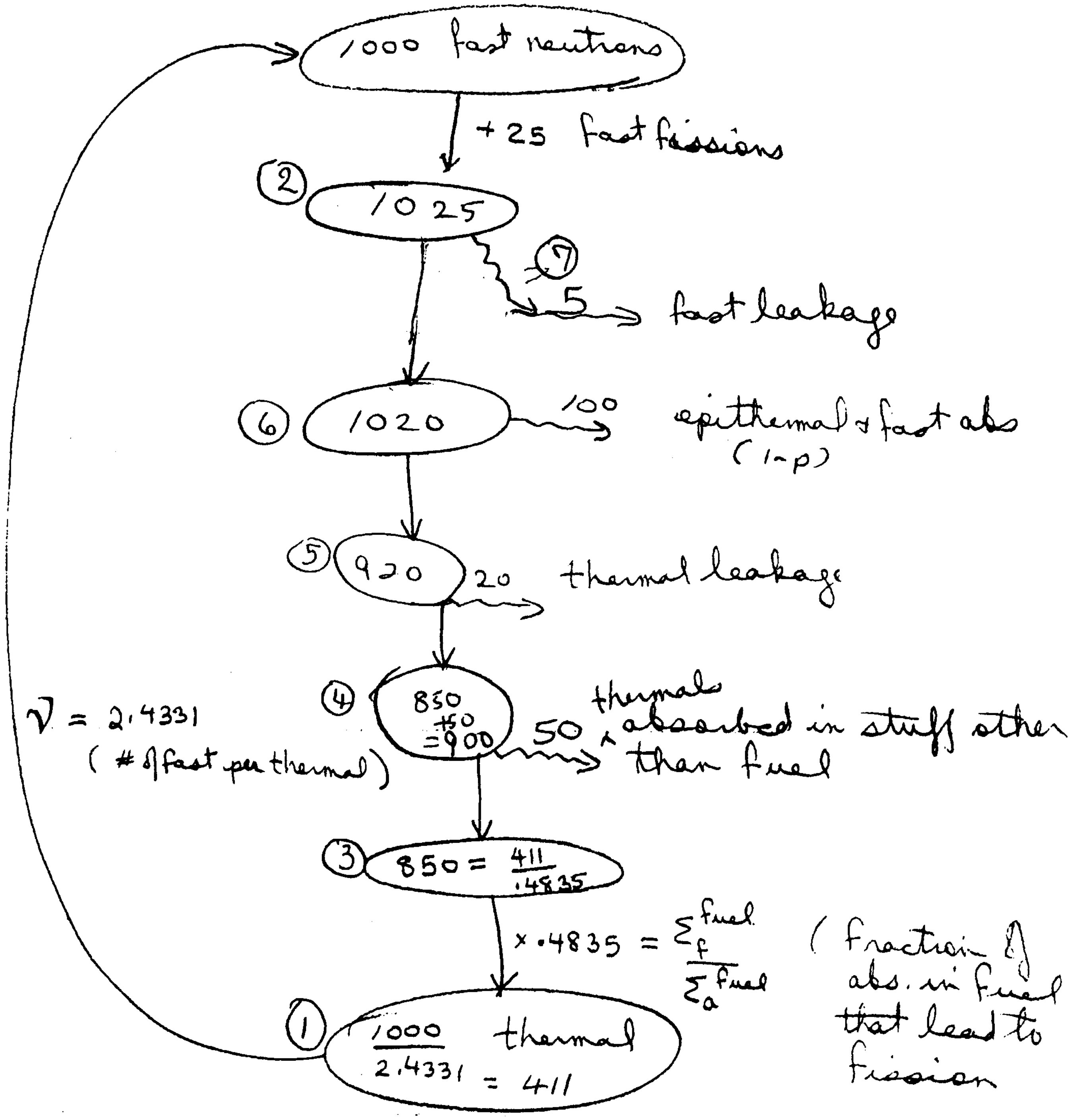
= # of fission neutrons produced / abs. in fuel $\quad \equiv$ thermal utilization

(e) $\rho \equiv \frac{K-1}{K}$, $K = \frac{\text{\# of neutrons in generation } n+1}{\text{" " " " } n}$

$= \frac{\text{Production rate}}{\text{Loss rate}}$

$\simeq 1$

$\therefore \rho \sim 0$


1000 fast neutrons
+25 fast fissions
2
1025
7
5 fast leakage
6
1020
100 epithermal & fast abs
(1−p)
5
920
20 thermal leakage
4
850
+50
=900
50 thermals absorbed in stuff other than fuel
ν = 2.4331
(# of fast per thermal)
3
850 = 411 / .4835
× .4835 = Σ_f^fuel / Σ_a^fuel
(fraction of abs. in fuel ~~that~~ lead to fission)
1
1000 / 2.4331 thermal = 411

2 (cont'd)

① If have 1000 fast neutrons & 2.4331 fast are produced for every thermal fission, then must have $\frac{1000}{2.4331}$ thermal fissions

② Add 25 fast neutrons due to fast fission.

③ Need $\frac{411}{.4835}$ thermal neutrons abs. in fuel if only .4835 (as a fraction) ~~lead~~ to fission.

$= 850$

④ 50 lost in non-fuel abs. ∴ must have had $850 + 50 = 900$ thermals.

⑤ Further 20 thermal leak ∴ must have had 920 (= 900+20) ~~before that~~

⑥ 100 non-thermals are abs. ∴ must have had 1020 before that loss.

⑦ ~~Deduce~~ that 5 must have leaked out as fast neutrons.

3.

$$\phi = \frac{SL}{2D} e^{-x/L}$$

Absorption rate at $x = \Sigma_a \phi(x)$

Total abs. rate for $x > 0 = \int_0^\infty \Sigma_a \phi(x) dx$

$$= \frac{\Sigma_a SL}{2D} \int_0^\infty e^{-x/L} dx = \frac{\Sigma_a SL}{2D} \cdot (-L) \, e^{-x/L} \Big|_0^\infty$$

$= \frac{S}{2}$ = production rate for R.H.S

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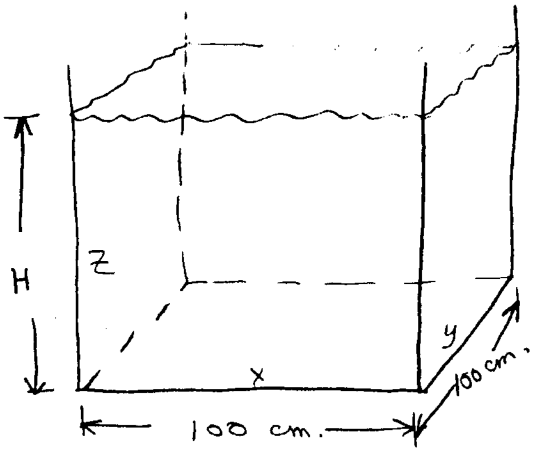
(a) $\Sigma_a = 0.500\ cm^{-1}$

$D = 10\ cm$.



$$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\nabla^2\phi + (\nu\Sigma_f - \Sigma_a)\phi$$

S.S: $D\nabla^2\phi + (\nu\Sigma_f - \Sigma_a)\phi = 0$

$$\phi = \phi_0 \cos\alpha x \ \cos\beta y \ \cos\gamma z$$

$$\Downarrow$$

$$-D(\alpha^2 + \beta^2 + \gamma^2) + (\nu\Sigma_f - \Sigma_a) = 0 \Leftarrow \text{Criticality Condition}$$

& Boundary Conditions $\Rightarrow \alpha = \frac{\pi}{100}, \ \beta = \frac{\pi}{100}, \ \gamma = \frac{\pi}{H}$

$\therefore \nu\Sigma_f = \Sigma_a + D\left(\frac{3\pi^2}{(100)^2}\right) = 0.500 + 0.0296 = \boxed{.5296\ cm^{-1}}$

(a) $= \nu\Sigma_f$

b) With absorber (which doesn't displace volume)

$$-D\left(\left(\frac{\pi}{100}\right)^2 + \left(\frac{\pi}{100}\right)^2 + \left(\frac{\pi}{110}\right)^2\right) + \nu\Sigma_f - \Sigma_a - \Sigma_{abs} = 0$$

$\uparrow$ additional Σ_a due to absorber

$$\therefore \Sigma_{abs} = -10\pi^2\left[\frac{2}{100^2} + \frac{1}{110^2}\right] + 0.0296$$

$$= -0.0279 + 0.0296$$

$$\boxed{\Sigma_{abs} = 0.0017\ cm^{-1}}$$

#5

2-group ; $\chi_1 = 1$ $\quad \Sigma_{s_{21}} = 0$ (no upscatter)
$\chi_2 = 0$ $\quad$ Homogeneous. $\quad \Sigma_{f_1} \simeq 0$

$\therefore$ in steady state:

$$-D_1 \frac{\partial^2 \phi_1}{\partial x^2} + \Sigma_{R_1} \phi_1 = \nu_2 \Sigma_{f_2} \phi_2$$

$$-D_2 \frac{\partial^2 \phi_2}{\partial x^2} + \Sigma_{a_2} \phi_2 = \Sigma_{s_{12}} \phi_1$$

For bare reactor, ϕ_1 & ϕ_2 have same shape (cosine)
Usual B.C.: $\phi(\pm a/2) = 0$ (at extrapolated distance)

Let $\phi_1(x) = \not{\phi}_1 \psi(x)$, $\phi_2(x) = \not{\phi}_2 \psi(x)$

st $\nabla^2 \psi + B^2 \psi = 0$

$$\therefore \begin{bmatrix} D_1 B^2 + \Sigma_{R_1} & -\nu_2 \Sigma_{f_2} \\ -\Sigma_{s_{12}} & D_2 B^2 + \Sigma_{a_2} \end{bmatrix} \begin{bmatrix} \not{\phi}_1 \\ \not{\phi}_2 \end{bmatrix} = 0$$

ie $\underline{\underline{A}}\, \underline{\phi} = 0$

Which only has a non-trivial solution if $|\underline{A}| = 0$

$$\therefore \boxed{(D_1 B^2 + \Sigma_{R_1})(D_2 B^2 + \Sigma_{a_2}) - \nu_2 \Sigma_{f_2} \Sigma_{s_{12}} = 0}$$

criticality condition

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$$\rho C_p \frac{\partial T}{\partial t} = q''' \leftarrow \text{heat generation rate}$$

$$\therefore \quad \rho C_p \Delta T \simeq q''' \Delta t$$

$$\therefore \quad \underbrace{\rho V}_{\text{Mass}} C_p \Delta T \simeq \underbrace{q''' V \Delta t}_{\text{total heat generated}} = Q \quad , \quad V = \text{volume.}$$

For a mixture of Al & U, the ΔT is the same for both & the Q is distributed into the combined homogeneous mass, ie:

$$(M_{Al} C_{p_{Al}} + M_u C_{p_u}) \Delta T = Q$$

$$\therefore \quad \Delta T = \frac{Q}{M_{Al} C_{p_{Al}} + M_u C_{p_u}}$$

$$= \frac{2.2 \times 10^6 \text{ J}}{(24.76 \times 903.5 + 7.539 \times 201.6) \text{ J/}^\circ\text{K}}$$

$$= 92.09 \ ^\circ\text{K}$$

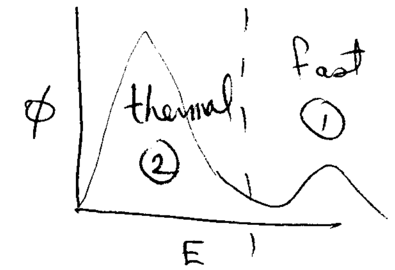
#7 a) – 2 group approximation $\therefore G=2$

– no upscatter $\therefore \Sigma_{s21}=0$

– no fast fissions $\therefore \Sigma_{f_1}=0$

– no thermal births $\therefore \chi_2=0 \Rightarrow \chi_1=1$

$\chi_2^c=0 \Rightarrow \chi_2^1=1$



$$\therefore \frac{1}{v_1}\frac{\partial\phi_1}{\partial t} = \nabla\cdot D_1\nabla\phi_1 - \underbrace{\Sigma_{a_1}\phi_1 - \Sigma_{s_1}\phi_1 + \Sigma_{s_{11}}\phi_1}_{=-\Sigma_{R_1}} + \cancel{\Sigma_{s_{21}}\phi_2}^{\,0}$$

$$+ (1-\beta)\nu_2\Sigma_{f_2}\phi_2 + \sum_{i=1}^{6}\lambda_i C_i + S_1^{ext}$$

forget this for convenience. No relevant to the issues at hand.

$$\frac{1}{v_2}\frac{\partial\phi_2}{\partial t} = \nabla\cdot D_2\nabla\phi_2 - \underbrace{\Sigma_{a_2}\phi_2 - \Sigma_{s_2}\phi_2 + \Sigma_{s_{22}}\phi_2}_{\equiv -\Sigma_{R_2}\phi_2} + \Sigma_{s_{12}}\phi_1$$

$$+ 0 + 0 + S_2^{ext}$$ ← ignore.

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i \nu_2 \Sigma_{f_2}\phi_2$$

b) S.S. ~~Iodine~~: $0 = \gamma_I \Sigma_{f_2}\phi_2 - \lambda_I I$

$$\Rightarrow I_\infty(r) = \frac{\gamma_I \Sigma_{f_2}\phi_2(r)}{\lambda_I}$$

fuel regions only

S.S. Xenon: $0 = \gamma_X \Sigma_{f_2}\phi_2 + \lambda_I I - \lambda_X X$

$$- X\underbrace{(\sigma_{a_1}^X\phi_1 + \sigma_{a_2}^X\phi_2)}_{\equiv\langle\sigma_{a_2}\phi\rangle}$$

$$\therefore X_\infty(r) = \frac{(\gamma_X+\gamma_I)\Sigma_{f_2}\phi_2(r)}{\lambda_X + \langle\sigma_{a_2}\phi\rangle}$$

fn(r)

These poisons affect the Σ_a coefficients mainly & to a lesser extent D & Σ_s

#7 (cont'd)

c) <u>SS C_i</u>

$$0 = -\lambda_i C_i + \beta_i \nu_2 \Sigma_{f_2} \phi_2$$

$$\therefore C_i(\vec{r}) = \frac{\beta_i \nu_2 \Sigma_{f_2} \phi_2(r)}{\lambda_i}$$

fuel regions <u>only</u>.

d) Plugging in to flux equations of (a):

$$\frac{1}{v_1}\cancelto{0}{\frac{\partial \phi_1}{\partial t}} = \nabla\cdot D_1 \phi_1 - \Sigma_{R_1}\phi_1 + (1-\beta)\nu_2 \Sigma_{f_2}\phi_2 + \sum_{i=1}^{6} \frac{\lambda_i \beta_i \nu_2 \Sigma_{f_2} \phi_2}{\lambda_i}$$

$$= \nabla\cdot D_1 \phi_1 - \Sigma_{R_1}\phi_1 + \nu_2 \Sigma_{f_2}\phi_2 \qquad \text{since } \beta = \sum_{i=1}^{6} \beta_i$$

$$\frac{1}{v_2}\cancelto{0}{\frac{\partial \phi_2}{\partial t}} = \nabla\cdot D_2 \nabla \phi_2 - \Sigma_{R_2}\phi_2 + \Sigma_{S_{12}}\phi_1 \quad \text{(unchanged from before)}$$

e) The S.S. fudge factor was only introduced as a means of artificially finding that particular combination of variables that precisely balanced the neutron gains and losses. The term in the flux equation that was most uncertain <u>and</u> affected the balance significantly was the $\nu \Sigma_f \phi$ term. Hence the term was fudged to $\frac{\nu \Sigma_f \phi}{K}$ and K was adjusted to give criticality. We could have adjusted Σ_a just as easily.

For the transient, we want to track the dynamic response to perturbations, controller actions and the like as poisons grow in, etc. So it is best to use a realistic controller that mimics the actual control rods.