1. - missing

Final 1996

2. a) The 1 speed eqn is:

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = \nabla \cdot D \nabla \phi - \Sigma_a \phi + \nu \Sigma_f \phi$$

in steady state: $\nabla \cdot D \nabla \phi - \Sigma_a \phi + \nu \Sigma_f \phi = 0$

or $\nabla \cdot D \nabla \phi - \Sigma_a \phi = -\nu \Sigma_f \phi$

so the signs of the terms are incorrect.

<u>And</u> $D = D(r)$ $\therefore$ can't take outside of ∇ operator.

$\therefore$ $D(r)\nabla^2\phi(r)$ should be $\nabla \cdot D(r) \nabla \phi$

b) $\Sigma_{removal} > \Sigma_a$ since removal includes outscattering & absorption.

c) The flux is continuous at an interface, not the current.

d) $\rho \equiv \frac{k-1}{k}$. $\quad k \in (0, \infty)$.

$\therefore$ ρ min is $\frac{-1}{0} = -\infty$

ρ max is $\frac{\infty}{\infty} = 1$

$\therefore$ ρ cannot be 2

e) at Power constant, as $\Sigma_{fuel} \downarrow$, $\phi \uparrow$.

3 a) A control rod is a neutron absorber. It controls the net rate of neutron production by altering the absorption rate, $\Sigma_{a_2}\phi_2$. Since the thermal X sections are relatively high, the thermal fluxes are most directly affected.

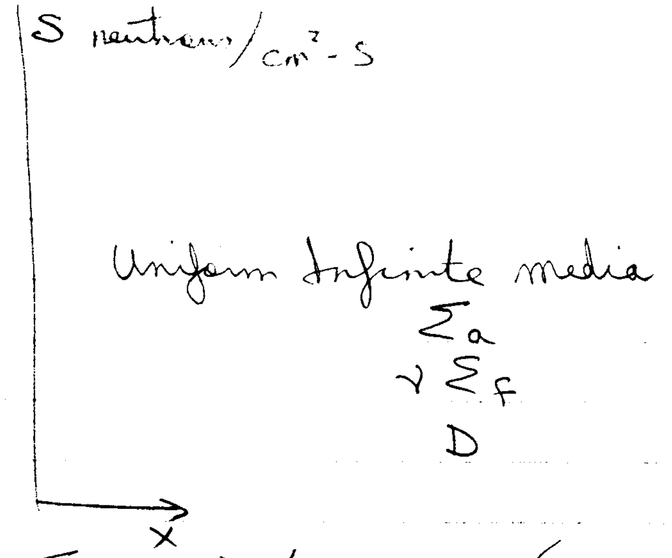
b) For maximum effectiveness, the rods should be placed in areas of maximum thermal flux. One large absorber would cause a massive flux depression, distorting the power production adversely (some fuel elements would be underutilized and others overutilized for a set average power), and lowering the effectiveness of the absorber ($\Sigma a \phi$ will drop if ϕ drops).
It is better to use many smaller rods. The flux shape can thus be optimized to get the most out of each fuel channel.

c) The delayed precursors slow down the flux transients so that rod movements with time constants of the order of 10's of msec. are sufficiently fast.

d) Yes, this will work. If $\phi_{meas} > \phi_{setpoint}$, the proportional term will effect a step change in the rod position (inward), increasing absorption and lowering flux. However, by the time the flux drops below the setpoint, the rod may be in too far. Hence, the rate term is usually necessary to be more proactive. It is responsive to the rate of flux change and will cause the rod to insert when the rate changes, even though the flux is still at the setpoint. Don't use thermal power. It responds too slowly.

e) Measure flux in a high flux region (near the core centre for best resolution + representation of the core behaviour) but stay away from control rods where the flux is distorted.

4.



$$D\frac{d^2\phi}{dx^2} + (\nu\Sigma_f - \Sigma_a)\phi = 0 \qquad (x \neq 0)$$

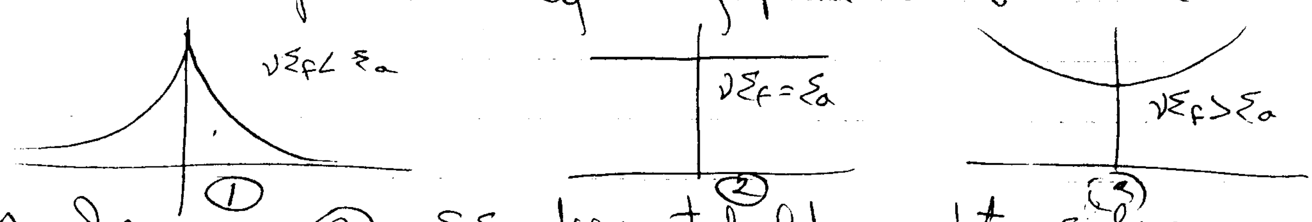
Compare this to case done in class for no fissile material.

$$\Rightarrow \quad D\frac{d^2\phi}{dx^2} - \Sigma_a\phi = 0 \qquad (x \neq 0)$$

This is the same except define $\boxed{L^2 = \frac{D}{\Sigma_a - \nu\Sigma_f}}$

(a) $\Rightarrow \boxed{\phi = \frac{SL}{2D} e^{-x/L}}$

(b) as $\nu\Sigma_f$ increases, $\Sigma_a - \nu\Sigma_f$ decreases and L increases + distribution flattens out. At some point, $\nu\Sigma_f = \Sigma_a$ + fission birth = absorption. At this point, any S neutron lives (effectively) forever (ie no net absorption). Beyond that (as $\nu\Sigma_f$ increases to be greater than Σ_a), the solution increases exponentially away from the source

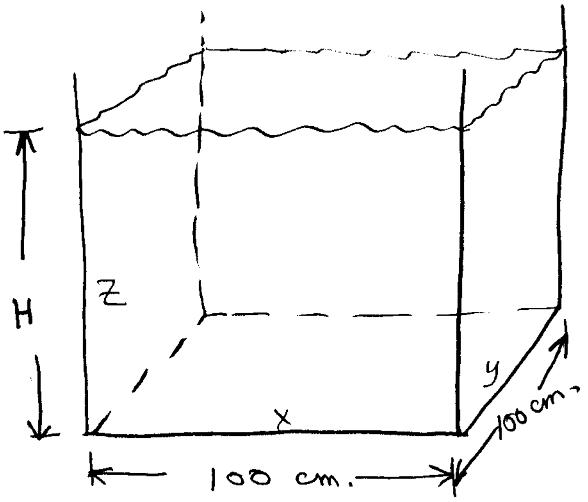


However, In case ③, S.S. does not hold – need to solve transient eqn $\Rightarrow$ runaway reactor.

5.

(a) $\Sigma_a = 0.500\ cm^{-1}$

$D = 10\ cm.$



$$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\nabla^2\phi + (\nu\Sigma_f - \Sigma_a)\phi$$

S.S: $D\nabla^2\phi + (\nu\Sigma_f - \Sigma_a)\phi = 0$

$$\phi = \phi_0 \cos\alpha x \ \cos\beta y \ \cos\gamma z$$

$$\Downarrow$$

$$-D(\alpha^2 + \beta^2 + \gamma^2) + (\nu\Sigma_f - \Sigma_a) = 0 \Leftarrow \text{Criticality Condition}$$

& Boundary Conditions $\Rightarrow$ $\alpha = \frac{\pi}{100}$, $\beta = \frac{\pi}{100}$, $\gamma = \frac{\pi}{H}$

$\therefore$ $\nu\Sigma_f = \Sigma_a + D\left(\frac{3\pi^2}{(100)^2}\right) = 0.500 + 0.0296 = \boxed{.5296\ cm^{-1} \quad (a)\ = \nu\Sigma_f}$

b) With absorber (which doesn't displace volume)

$$-D\left(\left(\frac{\pi}{100}\right)^2 + \left(\frac{\pi}{100}\right)^2 + \left(\frac{\pi}{110}\right)^2\right) + \nu\Sigma_f - \Sigma_a - \Sigma_{abs} = 0$$

additional Σ_a due to absorber

$$\therefore \Sigma_{abs} = -10\pi^2\left[\frac{2}{100^2} + \frac{1}{110^2}\right] + 0.0296$$

$$= -0.0279 + 0.0296$$

$$\boxed{\Sigma_{abs} = 0.0017\ cm^{-1}}$$

6. Given $n(t) = n_0 e^{-\alpha t} = e^{-0.01t}$

$\therefore n(t-\tau) = n_0 e^{-\alpha(t-\tau)}$

$\therefore \dfrac{n(t-\tau)}{n(t)} = e^{-\alpha\tau}$

$D(\tau) = \lambda e^{-\lambda\tau}$

$\lambda = .02 \text{ sec}^{-1}$
$\alpha = .01 \text{ sec}^{-1}$
$\Lambda = 5\times10^{-5} \text{ sec}$
$\beta = 0.007$

$$\therefore \rho = \beta + \Lambda \frac{\frac{d}{dt}\left(n_0 e^{-\alpha t}\right)}{n_0 e^{-\alpha t}} - \beta\int_0^\infty \lambda e^{-\lambda\tau} e^{\alpha\tau}\, d\tau$$

$$= \beta - \alpha\Lambda - \frac{\beta\lambda}{\alpha-\lambda}\, \underbrace{e^{(\alpha-\lambda)\tau}\Big|_0^\infty}_{0-1 \text{ since } \alpha<\lambda}$$

$$= \beta - \alpha\Lambda - \frac{\beta\lambda}{\lambda-\alpha}$$

$$= .007 - .01\times5\times10^{-5} - \frac{.007\times.02}{.01}$$

$$= -.007 - 5\times10^{-7} = -7.0005\times10^{-3}$$

$$\simeq \underline{\underline{-7 \text{ mk}}}.$$

Also could use inhour eqn $\rho = \dfrac{\omega l}{1+\omega l} + \dfrac{1}{1+\omega l}\displaystyle\sum_i \frac{\omega\beta_i}{\omega+\lambda_i}$

$\Lambda = \dfrac{l}{k} = l(1-\rho)$ $(\alpha = -\omega)$

or $\omega = \dfrac{\rho-\beta}{\Lambda} + \displaystyle\sum_i \frac{\lambda_i\beta_i}{\Lambda(\omega+\lambda_i)} = \dfrac{\rho-\beta}{\Lambda} + \dfrac{\lambda\beta}{\Lambda(\omega+\lambda)}$

$\therefore \omega\Lambda = \rho - \beta + \dfrac{\lambda\beta}{(\omega+\lambda)}$

$\therefore \rho = \beta + \omega\Lambda - \dfrac{\lambda\beta}{(\omega+\lambda)}$ ← same as above.

7. (10 marks)

From the notes we have: $q' = q_o' \cos\left(\frac{\pi z}{H}\right)$

where $z \in (-H/2, H/2)$

Coolant

$$T_{fluid}(z) = T_{inlet} + \frac{q_o' H}{\pi C_p W}\left[\sin\left(\frac{\pi z}{H}\right) + 1\right]$$

$$z \in (-H/2, H/2)$$

Fuel

$$\not\!\!\!T_{\not{c}} = T_{fluid}(z) + \frac{q'(z)}{2\pi r_f}\left[\frac{r_f}{2\bar{k}_f} + \frac{1}{h_G} + \frac{t_c + t_G}{k_c} + \frac{r_f}{h_s(r_f + t_c + t_G)}\right]$$

Since no sheath, $t_c = t_G = 0$, $h_G = \infty$

$$\therefore T_{\not{c}}(z) = T_{fluid}(z) + \frac{q'(z)}{2\pi r_f}\left[\frac{r_f}{2\bar{k}_f} + \frac{1}{h_s}\right]$$

$$= T_{inlet} + \frac{q_o' H}{\pi C_p W}\left[\sin\left(\frac{\pi z}{H}\right) + 1\right] + \frac{q_o' \cos\left(\frac{\pi z}{H}\right)}{2\pi r_f}\left[\frac{r_f}{2\bar{k}_f} + \frac{1}{h_s}\right]$$

8

a) S.S. $\Rightarrow$ precursors cancel out & are irrelevant to the flux eqn:

$$\Rightarrow -\nabla \cdot D_g \nabla \phi + \Sigma_{ag}\phi_g + \Sigma_{sg}\phi_g = \sum_{g'=1}^{G} \Sigma_{sg'g}\phi_g +$$

$$= \chi_g \sum_{g'=1}^{G} \nu_{g'}(1-\beta_{g'})\Sigma_{fg'}\phi_{g'} + S_g^{ext}$$

$\Rightarrow$ see end of answer

Poisons are at equilibrium at S.S:

$$X_\infty = \frac{(\gamma_I + \gamma_x)\,\Sigma_F \phi_0}{\lambda_x + \sigma_a^x \phi_0} \qquad (\text{from S.S. sol'n of } I + X \text{ eqn's})$$

This Xe concentration gives part of Σ_{ag} for the fuel regions $\Rightarrow$ $\Sigma_{ag} = \Sigma_{ag}(\text{other stuff}) + \sigma_{Xe} X_\infty$

Assume fuel at some level. ∴ get S.S. ϕ for various fuel loadings. Fuel changes slowly (days $\Rightarrow$ months)

b) Xe transient: After a short startup transient, the flux & delayed precursors settle down to a solution as in (a) but $X = I = 0$ at the start, ∴ no poisons at early times.

Solve the X & I equations numerically given the SS. flux. Every so often, say 5 minutes of simulation time, recalculate $\phi(r)$ with Σ_{ag} adjusted to account for Xe buildup.
Ignore fuel depletion for this timeperiod.

c) Short term transients (few minutes):
Xe + I don't change, fuel doesn't change.

Solve only the flux + precursor equations. Since the neutron velocity is high even for slow neutrons, the flux quickly sorts itself out, long before the precursors change. Therefore, solve for S.S. flux only, given the precursor concentration.

Solve the precursor eqn's numerically + update the SS. flux equations every few seconds.

d) fast transients (few seconds):
The precursors don't change much ∴ ignore them. Start from a SS. flux (where precursors are not a factor - as in part (a)). Introduce a perturbation in Σ_f or Σ_a to initiate a transient. ~~Solve~~:

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} = \nabla \cdot D_g \nabla \phi_g - \Sigma_{ag} \cdots$$

~ to part (a) but transient.

Ignore all other equations.

a) cont'd For 2 group, $g = 1, 2$ (fast, thermal)
for no upscatter, $\Sigma_{S21} = 0$, $\Sigma_{S22} = \Sigma_{S2}$
for no fast fissions, $\nu_1 = 0$
for no thermal births $\chi_2 = 0$

combine to $\Sigma_{r1}\phi_1$

$$\therefore \frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = \nabla \cdot D_1 \nabla \phi_1 + \Sigma_{a1}\phi_1 + \Sigma_{S1}\phi_1 = \Sigma_{S11}\phi_1 + \Sigma_{S12}\phi_2 + \nu_2(1-\beta)\phi_2 + \sum_i \lambda_i C_i$$

$$\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = \nabla \cdot D_2 \nabla \phi_2 + \Sigma_{a2}\phi_2 + \cancel{\Sigma_{S2}\phi_2} = \Sigma_{S12}\phi_1 + \cancel{\Sigma_{S22}\phi_2} + 0$$