4D3 Solutions Final 95.

1. (a) $-\Sigma_a \longrightarrow +\Sigma_a$ 2

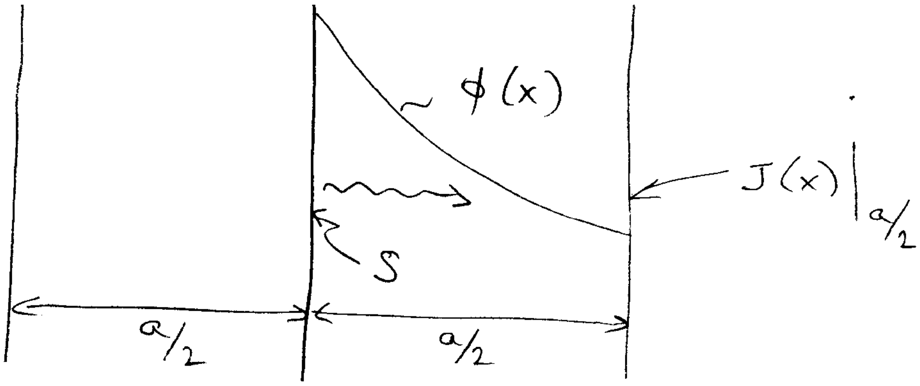
(b) half lives are reversed 2

(c) current is not *defined* as $-D\nabla\phi$. That is an approximation. 2

(d) If $\rho > \beta$, then superprompt critical. Therefore, the period is msec, not seconds. 2

(e) Power $= w\int_{V} \Sigma \phi \, dV$, $\therefore$ flux must $\uparrow$ if $V \downarrow$ 2

2.



Soln:

a) Probability of going from $x=0$ to $x=a/2$ without interaction is given by:

$$I(x) = I(0)\,e^{-\Sigma_t x} \Rightarrow \text{Prob} = e^{-\Sigma_t x}\Big|_{x=a/2}$$

$$\boxed{\text{prob} = e^{-\Sigma_t a/2}}$$

since this is just like a neutron beam being attenuated in a target. Any collision puts the neutron out of the running since we want only the ones ~~that~~ have not interacted at all.

b) The actual leakage out the edge is given by the current, $J(x)\big|_{a/2}$. The fraction or probability is $\dfrac{J(x)\big|_{a/2}}{S/2}$.

Now we know the flux is $\phi(x) = \dfrac{SL}{2D}\,\dfrac{\sinh[(a-2x)/2L]}{\cosh(a/2L)}$

(recall: $\sinh x = \dfrac{e^{x}-e^{-x}}{2}$)

$$\therefore J(x)\big|_{a/2} = -D\frac{SL}{2D}\,\frac{\left[-\frac{1}{L}e^{(a-\cancel{2}x)/2L} - \frac{1}{L}e^{0}\right]}{\cosh(a/2L)} = \frac{S}{2}\cdot\frac{1}{\cosh(a/2L)}$$

$$\therefore \boxed{\text{prob} = \frac{1}{\cosh(a/2L)}}$$

3. (a) In steady state:

$$0 = \nabla \cdot D_g \nabla_g - \Sigma_{a_g}\phi_g - \Sigma_{sg}\phi_g + \sum_{g'=1}^{G} \Sigma_{s_{g'g}}\phi_{g'} \quad ①$$

$$+ \chi_g \left[\sum_{g'=1}^{G} \nu_{g'}(1-\beta_{g'})\Sigma_{f_{g'}}\phi_{g'} + \sum_{i=1}^{N} \lambda_i C_i \right] + S_g^{ext}$$

and

$$0 = -\lambda_i C_i + \sum_{g=1}^{G} \beta_{ig}\nu_g \Sigma_{f_g}\phi_g \quad ②$$

Subst. ② → ① & focussing on the terms in the [....], we find:

$$\sum_{g'=1}^{G} \nu_{g'}(1-\beta_{g'})\Sigma_{f_{g'}}\phi_{g'} + \sum_{g=1}^{G} \nu_g \Sigma_{f_g}\phi_g \sum_{i=1}^{N}\beta_{ig}$$

$$'' \ \beta_{ig}$$

$$= \sum_{g'=1}^{G} \nu_{g'} \Sigma_{f_{g'}}\phi_{g'}$$

∴ simplifies down to the usual S.S. eqn.
∴ Precursors play <u>No role</u> in the S.S. flux.

(4)

(b) Poison affects the Absorption and ~~Scattering~~ Xsections. As per the text & class notes

$$\Delta\rho \simeq -\frac{\Sigma_a^{P}}{\underbrace{\Sigma_a^{F} + \Sigma_a^{M}}_{\text{all else}}}$$

based on k_∞ considerations
poison Σ

(2)

Basically, any new material shows up in the Σ's as a function of $(\underline{r}, t)$.

3 (c) Fuel concentration will affect Σ_a Σ_s & $\nu\Sigma_f$ for those spatial regions that contain fuel. This is a slow process (days, weeks, months) compared to poison (hours) and precursors (seconds). (2)

(d) "directly coupled" means that scattering doesn't skip groups. Thus:

$$\sum_{g'=1}^{G}{}' \Sigma_{sgg'}\phi_{g'} \Rightarrow \sum_{g'=g-1}^{g+1} \Sigma_{sgg'}\phi_{g'}$$

(allowing for
1 upscatter
2 downscatter
3 in-group scatter) (2)

(e) "no upscatter" means neutrons cannot gain enough energy ~~upon~~ upon scattering to jump to a higher energy group.

$$\therefore \sum_{g'=1}^{G}{}' \Sigma_{sgg'}\phi_{g'} \rightarrow \sum_{g'=g-1}^{g}{}' \Sigma_{sgg'}\phi_{g'}.$$ (2)

4.

$$0 = \nabla \cdot D_g \nabla \phi - \Sigma_a \phi_g - \Sigma_{sg} \phi_g + \sum_{g'=1}^{3} \Sigma_{sgg'} \phi_{g'} + \chi_g \sum_{g'=1}^{3} \nu_{g'} \Sigma_{fg'} \phi_{g'}$$

no upscatter: $\rightarrow \sum_{g'=1}^{g} \Sigma_{sgg'} \phi_{g'}$

no fission neutrons born in thermal group

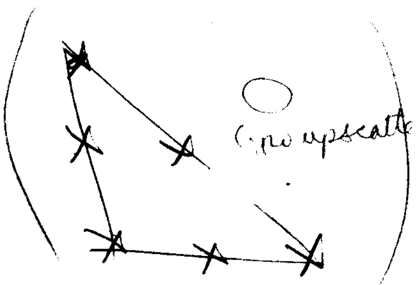
$\rightarrow \chi_3 = 0$

fission only in lowest 2 groups:

$\Rightarrow \nu_1 = 0$

$$\therefore \; -\nabla \cdot D_g \nabla \phi_g + \Sigma_{a_g} \phi_g + \Sigma_{sg} \phi_g - \sum_{g'=1}^{g} \Sigma_{sg'g} \phi_{g'} = \chi_g \sum_{g'=2}^{3} \nu_{g'} \Sigma_{fg'} \phi_{g'}$$

χ_g



$$\begin{pmatrix} \phi \end{pmatrix} = \begin{pmatrix} \chi_1 (\nu_2 \Sigma_{f_2} \phi_2 + \nu_3 \Sigma_{f_3} \phi_3) \\ \chi_2 (\nu_2 \Sigma_{f_2} \phi_2 + \nu_3 \Sigma_{f_3} \phi_3) \\ 0 \end{pmatrix}$$

$\nu_1 = 0$ (no fast fissions)

no fission births in thermal group

$$-\nabla \cdot D_1 \nabla \phi_1 + \underbrace{\Sigma_{a_1} \phi_g + \Sigma_{s_1} \phi_1 - \Sigma_{s11} \phi_1}_{\Sigma_{r_1} \phi_1} = \chi_1 (\nu_2 \Sigma_{f_2} \phi_2 + \nu_3 \Sigma_{f_3} \phi_3)$$

$$-\nabla \cdot D_2 \phi_2 + \underbrace{\Sigma_{a_2} \phi_2 + \Sigma_{s_2} \phi_2 - \cancel{\Sigma_{s12} \phi_1} - \Sigma_{s22} \phi_2}_{\Sigma_{r_2} \phi_2} = \chi_2 (\nu_2 \Sigma_{f_2} \phi_2 + \nu_3 \Sigma_{f_3} \phi_3) + \Sigma_{s12} \phi_1$$

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$$-\nabla \cdot D_3 \phi_3 + \underbrace{\Sigma_{a_3} \phi_3 + \cancel{\Sigma_{s_3} \phi_3}}_{\Sigma_{r_3} \phi_3} - (\Sigma_{s13} \phi_1 - \Sigma_{s23} \phi_2) - \cancel{\Sigma_{s33} \phi_3} = 0$$

since no upscatter $+$ $'' \Sigma_{s_3}$

ie $-\nabla \cdot D_3 \phi_3 + \Sigma_{a_3} \phi_3 = \Sigma_{s13} \phi_1 + \Sigma_{s23} \phi_2$

5. from inhour equation:

$$\rho = \frac{\omega l}{(1+\omega l)} + \frac{1}{(1+\omega l)} \frac{\omega \beta}{\omega+\lambda}$$

$T_{1/2} = 20$ sec.
$\therefore \lambda = \frac{\ln 2}{20}$

$l = 5 \times 10^{-5}$
$\beta = 0.007$
$\omega = \frac{1}{T} = \frac{1}{1}$ sec^{-1}

$$\therefore \rho = \frac{1 \times 5\times10^{-5}}{1 + 5\times10^{-5}} + \frac{1}{1+5\times10^{-5}} \times \frac{0.007}{1 + \left(\frac{\ln 2}{20}\right)}$$

$$\simeq 5\times10^{-5} + \frac{.007}{1+0.03466} = 0.00682$$

$\therefore \rho = 6.82$ mk

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6.

$$\frac{1}{v}\cancel{\frac{\partial \phi}{\partial t}}_{0} = D\nabla^2\phi - \Sigma_a\phi + (1-\beta)\nu\Sigma_f\phi + \sum_{i=1}^{6}\lambda_i C_i$$

$$\cancel{\frac{\partial C_i}{\partial t}}_{0} = -\lambda_i C_i(r,t) + \beta_i \nu \Sigma_f \phi$$

$$\therefore \quad C_i(r) = \frac{\beta_i \nu \Sigma_f \phi(r)}{\lambda_i}$$

$\therefore$ C_i has same shape as the flux.

Substituting C_i into flux eqn we get :

$$0 = D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi ,$$

ie the same as before.

In ~~an~~ homogeneous slab reactor, $\phi = \phi_0 \cos\left(\frac{\pi x}{a}\right)$

$$\therefore \quad C_i = \frac{\beta_i \nu \Sigma_f \phi_0}{\lambda_i} \cos\left(\frac{\pi x}{a}\right)$$

7. Given $n(t) = n_0 e^{-\alpha t}$

$\therefore\ n(t-\tau) = n_0 e^{-\alpha(t-\tau)}$

$\therefore\ \dfrac{n(t-\tau)}{n(t)} = e^{\alpha\tau}$

$D(\tau) = \lambda e^{-\lambda\tau}$

$$\therefore\ \rho = \beta + \Lambda \frac{\frac{d}{dt}(n_0 e^{-\alpha t})}{n_0 e^{-\alpha t}} - \beta\int_0^\infty \lambda e^{-\lambda\tau} e^{\alpha\tau}\,d\tau$$

$$= \beta - \alpha\Lambda - \frac{\beta\lambda}{(\alpha-\lambda)}\underbrace{e^{(\alpha-\lambda)\tau}\Big|_0^\infty}_{=\,0-1 \text{ if } \alpha<\lambda}$$

$$\therefore\ \boxed{\rho = \beta - \alpha\Lambda - \frac{\beta\lambda}{\lambda-\alpha}}$$

$$= \frac{\beta(\cancel{\lambda}-\alpha) - \cancel{\beta\lambda}}{\lambda-\alpha} - \alpha\Lambda$$

$$\boxed{\rho = \frac{-\alpha\beta}{\lambda-\alpha} - \alpha\Lambda}$$

$\therefore$ QED

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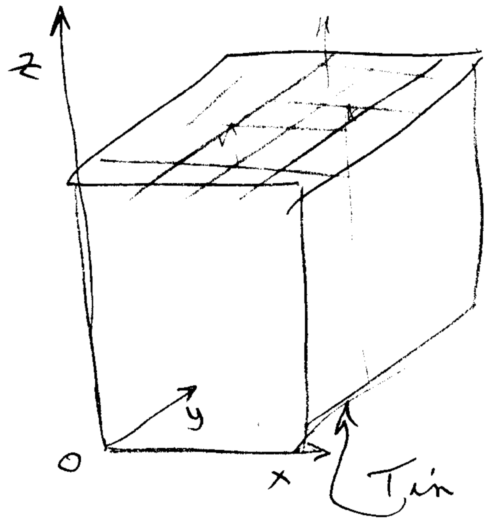
8 (a)



$$-D\nabla^2\phi + \Sigma_a\phi = \frac{\nu\Sigma_f\phi}{k} \quad (1)$$

$$\nabla^2\phi \equiv \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

W kg/s per fuel channel.

$$W\frac{\partial h}{\partial z} = q'(z) \Rightarrow \frac{\partial T}{\partial z} = \frac{1}{WC_p}q'(z) \quad (2)$$

heatflux perchannel perunit axial length.

$$\equiv \gamma\,\phi(x,y,z)$$

(b) BC: $\phi = 0$ at surface of reactor.

$$T(x,y,0) = T_{in}$$

Power known

(c) $$-D_{ijk}\left(\frac{\phi_{i+1jk} - 2\phi_{ijk} - \phi_{i-1jk}}{\Delta x^2}\right) + \sim \text{for } j+k$$

$$+ \Sigma_{a_{ijk}}\phi_{ijk} = \frac{\nu_{ijk}\Sigma_{F_{ijk}}\phi_{ijk}}{k}$$

$$\frac{T_{ijk+1} - T_{ijk}}{\Delta z} = \frac{1}{WC_p}\gamma\left(\frac{\phi_{ijk} + \phi_{ijk+1}}{2}\right)$$

d) Solve for flux as usual.
loop on ijk until converged

$$\phi_{ijk} = \frac{1}{\left[2D_{ijk}\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}+\frac{1}{\Delta z^2}\right)+\Sigma_{a_{ijk}}\right]}\left[\frac{D_{ijk}}{\Delta x^2}\left(\phi_{i+1,jk}+\phi_{i-1,jk}\right) + \frac{D_{ijk}}{\Delta y^2}\left(\phi_{ij+1k}\; , \right. \;\; , \;\; , \;\; , \;\; + \frac{\nu\Sigma_{f_{ijk}}}{K}\phi_{ijk}\right]$$

Then sweep up each channel k for all i & j

$$T_{ijk+1} = \frac{\Delta z}{W C_p}\gamma\left(\frac{\phi_{ijk}+\phi_{ijk+1}}{2}\right) + T_{ijk}$$