EP 403
Final Exam 1991

1. (15 marks)

a. (5 marks)

Boron is a good absorber, ∴ safe to assume negligible scattering (no buildup).

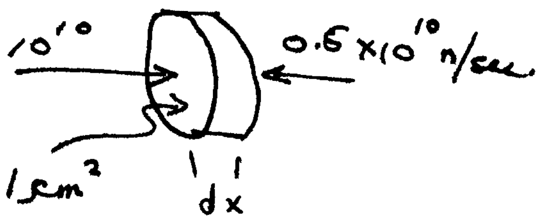
$$\therefore \quad \frac{I(x)}{I(0)} = 0.001 = e^{-\Sigma_a x}$$

$$\therefore \quad x = -\frac{\ln(0.001)}{\Sigma_a} = \frac{+6.91}{103}$$

Σ_a ↖ $103\ \text{cm}^{-1}$

$$= \underline{\underline{0.0671\ \text{cm}.}}$$

b. (10 marks)



= length swept by neutrons in 1 sec. = 2.2×10^5 cm. (assume thermal)

$$\phi = \frac{\int_V n v \, dV}{\int_V dV}$$

$$= \frac{1.5 \times 10^{10} \times 2.2 \times 10^5}{2.2 \times 10^5}$$

$$= 1.5 \times 10^{10} \ \frac{\text{neutrons}}{\text{cm}^2\text{-sec.}}$$

$$J = \int_S n \underline{v} \cdot d\underline{s} = \left(1.0 \times 10^{10} - 0.5 \times 10^{10}\right) \cdot 1\ \text{cm}^2$$

$$= 0.5 \times 10^{10}\ \hat{x}\ \text{neutrons/cm}^2\text{-sec.}$$

(positive x direction)

2.
(10 marks total)

$$\frac{dN}{dt} = -\lambda N - cN + R$$

$$= -(\lambda + c)N + R$$

(5 marks for correct rate eqn.)

Let $x(t) = N(t) - R \cdot a$

$\uparrow$ a constant to be determined

$$\therefore \frac{dx}{dt} = -(\lambda+c)\left[x + aR\right] + R$$

Let $a = \frac{1}{\lambda + c}$ to give

$$\frac{dx}{dt} = -(\lambda + c)\,x$$

$$\Rightarrow \quad x = x_0\, e^{-(\lambda+c)t}$$

(3 marks for a correct sol'n procedure)

where $x_0 = N(0) - \frac{R}{\lambda+c}$

$$\therefore N(t) = \left[N(0) - \frac{R}{\lambda+c}\right] e^{-(\lambda+c)t} - \frac{R}{\lambda+c}$$

$$\therefore \boxed{N(t) = \underbrace{N(0)\, e^{-(\lambda+c)t}}_{\text{decay + removal}} + \underbrace{\frac{R}{\lambda+c}\left(1 - e^{-(\lambda+c)t}\right)}_{\text{production}}}$$

(2 marks for correct solution)

3. (10 marks)

a. $\phi = \frac{SL}{2D} e^{-x/L}$, $x > 0$

Absorption rate at $x = \Sigma_a \phi(x)$

Total absorption rate for $x > 0$:

$$= \int_0^\infty \Sigma_a \phi(x)\,dx \qquad \#/_{cm} \cdot \frac{1}{cm^2\text{-sec.}}$$

$$= \frac{\Sigma_a S L}{2D} \int_0^\infty e^{-x/L}\,dx$$

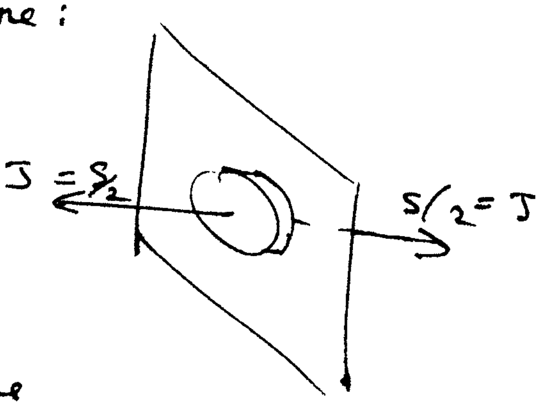
(4 marks for properly posing integral. 3 for proper solution)

$$= \frac{\Sigma_a S L}{2D} (-L) e^{-x/L}\Big|_0^\infty$$

$$= \frac{S}{2} \qquad \text{(7 marks)}$$

b. Current at source plane:

$$= \lim_{x \to \infty} J(x) = \frac{S}{2}$$

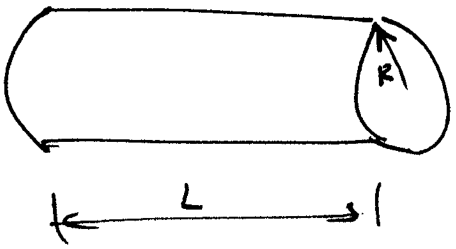


∴ As you might expect, the current flowing to the right equals exactly the absorption, ie sinks = sources in steady state. (3 marks)

4. (10 marks)

$$\nabla^2 \phi + B_g^2 \phi = 0 \Rightarrow B_g^2 = \left(\frac{\pi}{L}\right)^2 + \left(\frac{\nu_0}{R}\right)^2$$

for criticality



Volume $= \pi R^2 L$

For volume to be minimum: $\frac{\partial \text{Volume}}{\partial R} = \frac{\partial \text{Volume}}{\partial L} = 0$

Since L + R are constrained via B_g:

$$\frac{\nu_0}{R} = \sqrt{B_g^2 - \left(\frac{\pi}{L}\right)^2} \Rightarrow R = \frac{\nu_0}{\sqrt{B_g^2 - (\pi/L)^2}}$$

$$\therefore \text{Volume} = \pi R^2 L = \frac{\pi \nu_0^2 L}{\left(B_g^2 - (\pi/L)^2\right)}$$

$$\therefore \frac{\partial \text{Volume}}{\partial L} = \frac{\pi \nu_0^2}{[B_g^2 - (\pi/L)^2]} - \frac{\pi \nu_0^2 L \,(+2\pi^2/L^3)}{[B_g^2 - (\pi/L)^2]^2} = 0$$

$$\therefore 1 - \frac{2\pi^2}{L^2 [\underbrace{B_g^2 - (\pi/L)^2}_{(\nu_0/R)^2}]} = 0$$

$$\therefore \underset{B_g^2\ \text{radial}}{\left(\frac{\nu_0}{R}\right)^2} = \underset{B_g^2\ \text{axial}}{2\left(\frac{\pi^2}{L^2}\right)} \Rightarrow \text{Radial buckling} = 2 \times \text{axial buckling}$$

(6 marks for formulation of problem, 4 for solution)

5. (15 marks)

a. (4 marks)

$$-D_1^c \nabla^2 \phi_1^c + \Sigma_{R_1}^c \phi_1^c = \frac{1}{K}\left[\nu_1 \Sigma_{F_1}^c \phi_1^c + \nu_2 \Sigma_{F_2}^c \phi_2^c\right]$$

$$-D_2^c \nabla^2 \phi_2^c + \Sigma_a^c \phi_2^c = \Sigma_{S12}^c \phi_1^c$$

$$-D_1^R \nabla^2 \phi_1^R + \Sigma_{R_1}^R \phi_1^R = 0$$

$$-D_2^R \nabla^2 \phi_2^R + \Sigma_a^R \phi_2^R = \Sigma_{S12}^R \phi_1$$

b. (4 marks)

$$\phi_i^R(b/2) = 0 \qquad i = 1, 2$$

$$\phi_i^c(a/2) = \phi_i^R(a/2)$$

$$J_i^c(a/2) = J_i^R(a/2)$$

(ie $D_i^c \nabla\phi_i^c(a/2) = D_i^R \nabla\phi_i^R(a/2)$)

symmetry or $\nabla \phi_i^c(0) = 0$

c. (4 marks)

Try solutions of form: $\phi_i^{*} = A_i \cos\mu_i r + C_i \cosh\lambda_i r$ (or whatever, Bessel?) & subst. into eqn's in (a). This yields 4 eqn's that should allow the calc. of the buckling coefficients, μ_i + λ_i.

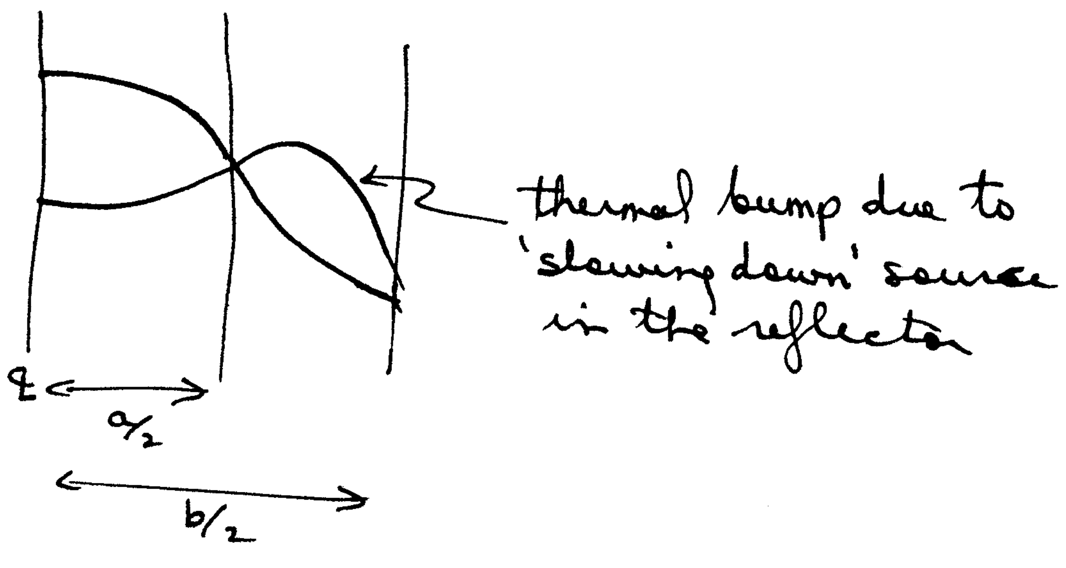
Next apply the B.C.'s. The condition

$\phi_i^R(a/2 + b) = 0$ should allow you to eliminate one unknown A_i^R or C_i^R for $i = 1,2$.

Symmetry does the same for the core flux.

This leaves 4 unknowns + 4 interface B.C. equations that can be solved (theoretically) for the 4 unknown. These 4 eqn's form the criticality condition.

d) (3 marks)

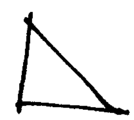


6. (15 marks)

a. Tightly Coupled: scattering down to nearest neighbour only
(5 marks)

$\Rightarrow$ 

No upscatter: can't scatter up

$\Rightarrow$ 

$$\Sigma_{s g' g} = 0 \text{ for } g' > g$$

$$\therefore \sum \Sigma_{s g g'} \phi_{g'} = \sum_{g'=1}^{g-1} \Sigma_{s g' g} \phi_g + \Sigma_{s g g} \phi_g$$

$\Rightarrow$ lump with Σ_{Rg}

$$\sum_{g'=1}^{G} \Sigma_{s g' g} \phi_{g'} = \Sigma_{s\, g-1, g} \phi_g + \Sigma_{s g g} \phi_g$$

b. As per text.
(5 marks)

eg
$$\Sigma_{ag} = \frac{\int_{E_g}^{E_{g-1}} \Sigma_a(E)\, \phi(E)\, dE}{\int_{E_g}^{E_{g-1}} \phi\, dE}$$

Thus
$$\Sigma_a = \sum_{g=1}^{G} \Sigma_{ag} \phi_g \Big/ \underbrace{\sum_{g=1}^{G} \phi_g}_{\approx\, \phi}$$

yields:

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = \nabla \cdot D \nabla \phi - \Sigma_a \phi \cancel{- \Sigma_s \phi_g} \cancel{+ \Sigma_s \phi_g}$$

$$+ \cancel{\quad} \nu \Sigma_f \phi + \sum_{i=1}^{N_I} \lambda_i C_i + S^{ext}$$

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i \nu \Sigma_f \phi$$

c. (5 marks)

If $t_{\frac{1}{2}}$ small, then λ_i large

$\therefore$ if C_i is significant, $\frac{\partial C_i}{\partial t}$ is large & -ve.

$\therefore$ precursors decay rapidly away.

$\therefore$ $C_i \sim 0$ & $\frac{\partial C_i}{\partial t} \sim 0$

$$\therefore \quad 0 = -\lambda_i C_i + \sum_{g=1}^{G} \beta_{ig} \nu_g \Sigma_{fg} \phi_g$$

$$\therefore \quad \sum_{i=1}^{N} \lambda_i C_i = \sum_{g=1}^{G} \nu_g \Sigma_{fg} \phi_g \underbrace{\sum_{i=1}^{N} \beta_{ig}}_{= \beta_g}$$

$\therefore$ term in flux eqn simplifies to:

$$\chi_g \sum_{g'=1}^{G} \nu_{g'} (1 - \beta_{g'}) \Sigma_{fg'} \phi_{g'} + \sum_{i=1}^{N} \lambda_i C_i$$

these 2 terms cancel.

& drop precursor eqn since $C_i \sim 0$

7. (10 marks)

From the notes we have: $q' = q_0' \cos\left(\frac{\pi z}{H}\right)$

where $z \in (-H/2, H/2)$

Coolant

$$T_{fluid}(z) = T_{inlet} + \frac{q_0' H}{\pi C_p W}\left[\sin\left(\frac{\pi z}{H}\right) + 1\right]$$

$$z \in (-H/2, H/2)$$

Fuel

$$\text{¢}\ T_{\text{¢}} = T_{Fluid}(z) + \frac{q'(z)}{2\pi r_f}\left[\frac{r_f}{2\bar{k}_f} + \frac{1}{h_G} + \frac{t_c + t_G}{k_c} + \frac{r_f}{h_s(r_f + t_c + t_G)}\right]$$

Since no sheath, $t_c = t_G = 0$, $h_G = \infty$

$$\therefore T_{\text{¢}}(z) = T_{Fluid}(z) + \frac{q'(z)}{2\pi r_f}\left[\frac{r_f}{2\bar{k}_f} + \frac{1}{h_s}\right]$$

$$= T_{inlet} + \frac{q_0' H}{\pi C_p W}\left[\sin\left(\frac{\pi z}{H}\right) + 1\right] + \frac{q_0' \cos\left(\frac{\pi z}{H}\right)}{2\pi r_f}\left[\frac{r_f}{2\bar{k}_f} + \frac{1}{h_s}\right]$$