4D3/6D3 Final Dec 2003

Solution

1. (a) $\frac{dn}{dt} = -\lambda n \Rightarrow dn = -\lambda n\, dt$

decay only.

(b) $\frac{dn}{dx} = -\Sigma_a n \Rightarrow dn = -\Sigma_a n\, dx$

absorption only

(c) $dn = -\lambda n\, dt - \Sigma_a n\, dx$

(d) ratio of decay rate to absorption rate

$$= \frac{\lambda n\, dt}{\Sigma_a n\, dx} = \frac{\lambda n\, dt}{\Sigma_a n \underbrace{v\, dt}_{dx \text{ since } x = vt}} = \frac{\lambda}{\Sigma_a v}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{11.7 \times 6} \text{ sec}^{-1} = 9.9 \times 10^{-4} \text{ sec}^{-1}$$

$v = 2.2 \times 10^5$ cm/sec.

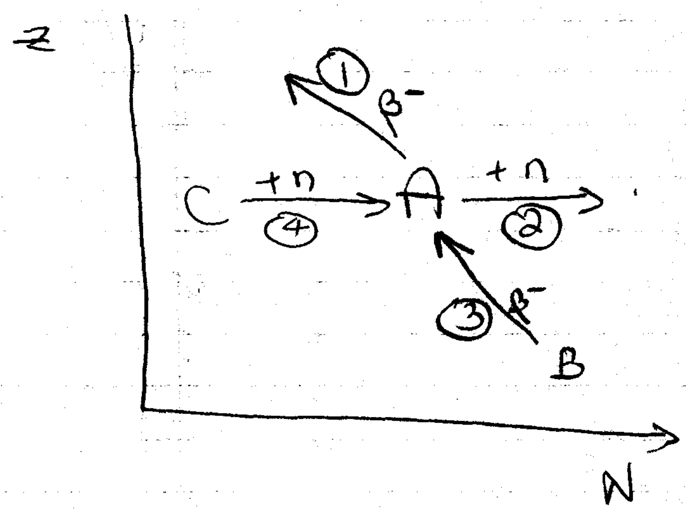
$\Sigma_a = 0.022 \text{ cm}^{-1}$

$$\therefore \text{ratio} = \frac{\lambda}{\Sigma_a v} = \frac{9.9 \times 10^{-4}}{0.022 \times 2.2 \times 10^5} = 2.04 \times 10^{-7}$$

ie decay is not likely

2.

$$\frac{dN_A}{dt} = \underset{\text{(decay)}}{\overset{①}{-\lambda_A N_A}} \; \underset{\text{(neutron capture)}}{\overset{②}{-\,\sigma_a^A \phi N_A}} \; \underset{\text{(decay of parent)}}{\overset{③}{+\,\lambda_B N_B}} \; \underset{\text{(capture transmutation)}}{\overset{④}{+\,\sigma_\gamma^C \phi N_C}}$$



3. From the inhour equation:

$$\rho = \frac{\omega l}{1+\omega l} + \frac{1}{1+\omega l} \frac{\omega \beta}{(\omega+\lambda)}$$

$l = 5\times10^{-5}$ sec. $\quad \beta = 0.007$

$\omega = \frac{1}{T} = \frac{1}{1}$ sec^{-1} $\quad \lambda = \frac{\ln 2}{20}$ sec^{-1}

$$\rho = \frac{1 \times 5\times10^{-5}}{(1+5\times10^{-5})} + \frac{1}{(1+5\times10^{-5})} \times \frac{.007}{1+\frac{\ln 2}{20}}$$

$$\simeq 5\times10^{-5} + \frac{.007}{1+0.03466} = 0.00682$$

$$\therefore \underline{\underline{\rho = 6.82 \text{ mk}}}$$

4. (a) $\frac{1}{v}\frac{\partial \phi}{\partial t} = D\nabla^2\phi + (\nu\Sigma_f - \Sigma_a)\phi + S$

steady state since concerned with eventual situation

since infinite: $\nabla^2\phi = 0$

S uniform

subcritical $\therefore \nu\Sigma_f < \Sigma_a$

$\therefore \phi = \frac{S}{\Sigma_a - \nu\Sigma_f}$ everywhere, Note: uniform in space.

Absorption rate $= \Sigma_a\phi = \frac{\Sigma_a S}{\Sigma_a - \nu\Sigma_f}$ = full production rate

Initial Production rate $= S$

$\therefore$ # produced per initial neutron

$$= \frac{\frac{\Sigma_a S}{(\Sigma_a - \nu\Sigma_f)}}{S} = \frac{\Sigma_a}{\Sigma_a - \nu\Sigma_f} = \frac{1}{1 - \nu\Sigma_f/\Sigma_a} \equiv M$$

Note that for 1 speed neutrons, $k_\infty \equiv \epsilon\eta f p$

$\simeq \eta f$

$\equiv \nu\Sigma_f/\Sigma_a$

$\therefore M = \frac{1}{1-k}$

(b) $k = \frac{\text{\# in generation } m+1}{\text{\# in generation } m}$

$\therefore$ tracking generations starting with, say, 100 neutrons:

$100 \rightarrow k \times 100 \rightarrow k^2 \times 100$ - - - -

ie total # $= 100\,(1 + k + k^2 + \ldots)$

$= 100\left(\frac{1}{1-k}\right)$

$\therefore M = \frac{1}{1-k}$ (as seen in (a))

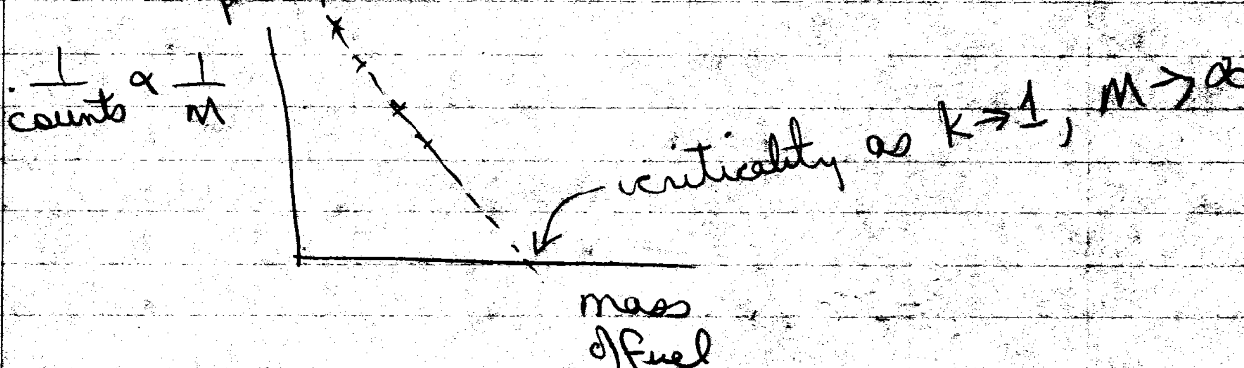
(c) as mentioned, $K_\infty = \underbrace{\epsilon}_{\sim 1} \eta f \underbrace{p}_{\sim 1,\text{ ie ignore}}$

$$\eta = \frac{\nu \Sigma_{f\,fuel}}{\Sigma_{a\,fuel}} \quad , \quad f = \frac{\Sigma_{a\,fuel}}{\Sigma_{a\,all}}$$

$$\therefore \quad K_\infty \simeq \frac{\nu \Sigma_{f\,fuel}}{\Sigma_{a\,all}} = \frac{\nu \Sigma_f}{\Sigma_a}$$ in the nomenclature of the problem.

$$\therefore \quad M_{part\,a} = \underline{\underline{\frac{1}{1 - \nu\Sigma_f/\Sigma_a} = \frac{1}{1-k}}} = M_{part\,b}$$

(d) If we use a neutron detector to measure counts (proportional to M) then we can plot

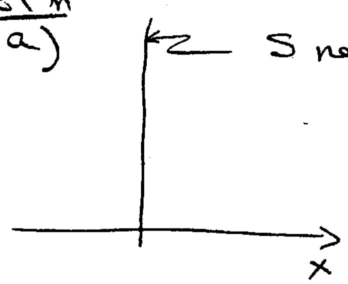


(e) We do not want to overshoot criticality since if $k \to 1+\beta$ ($= \sim 1.0065$), the reactor will be critical on prompt neutrons only whose time constant is ≤ 1 mS. This is to fast to control.

(5)

Sol'n

a)



$\leftarrow$ S neutrons / cm^2-sec

Uniform Infinite Media with fissile material

Σ_a

$\nu\Sigma_f$

D

$$D\frac{d^2\phi}{dx^2} + (\nu\Sigma_f - \Sigma_a)\phi = 0 \quad \text{for } x \neq 0$$

Compare this to case done in class for no fissile material:

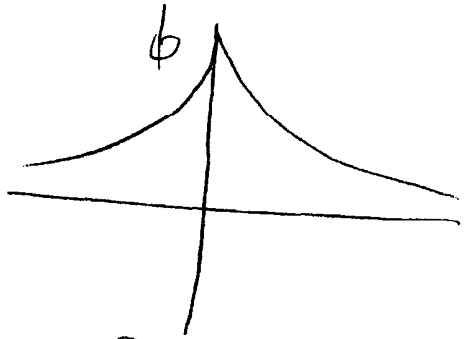
$$D\frac{d^2\phi}{dx^2} - \Sigma_a\phi = 0 \quad \text{for } x \neq 0.$$

This is the same except define $L^2 = D/\Sigma_a - \nu\Sigma_f$

$\therefore \quad \phi = \frac{SL}{2D} e^{-x/L}$ where $\uparrow$

b) Pile is initially subcritical, ie $\nu\Sigma_f$ is small cf Σ_a. As $\nu\Sigma_f \uparrow$, $(\Sigma_a - \nu\Sigma_f)\downarrow$ + thus $L\uparrow$. At some point $\nu\Sigma_f = \Sigma_a$ + fission birth = absorption. At this point any S neutron lives (effectively) forever (ie no net absorption). Beyond ~~that~~ (as $\nu\Sigma_f$ increases to be greater than Σa), the solution increases exponentially away from the source.

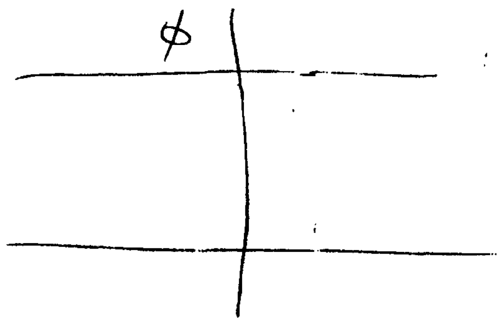
$\longrightarrow$ more



$$\phi = \frac{SL}{2D} e^{-x/L}$$

where $L > 0$

Case ① : $\nu \Sigma_f < \Sigma_a$



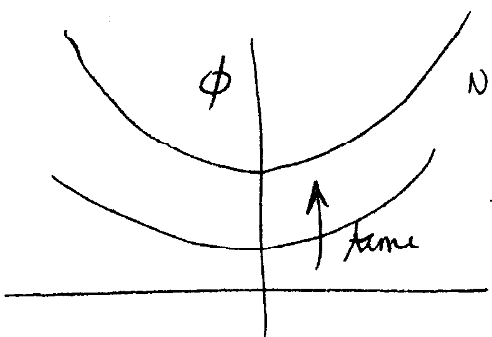
As $\nu \Sigma_f \to \Sigma_a$, $L \to \infty$

$$\phi \sim \frac{SL}{2D} e^{-x/L}$$

very slow decay in space.

Large increase in amplitude.

Case ② $\nu \Sigma_f = \Sigma_a$



Note: Steady state does not hold, ∴ need to solve transient equation for this runaway reactor.

Case ③ $\nu \Sigma_f > \Sigma_a$

6. (a) For: $\frac{1}{v}\cancel{\frac{\partial\phi}{\partial t}}_{0} = \nabla\cdot D\nabla\phi - \Sigma_a\phi + S$

simplify to $D\frac{\partial^2\phi}{\partial x^2}$ to illustrate

could be $\nu\Sigma_f\phi$ and fixed S

$$\therefore\ 0 = \frac{D}{\Delta x^2}\left(+\phi_W - 2\phi_P + \phi_E\right) - \Sigma_a\phi_P + S_P$$

$$\therefore\ \phi_P = \frac{1}{\left(\Sigma_a + \frac{2D}{\Delta x^2}\right)}\left[S_P + \frac{D}{\Delta x^2}\left(\phi_W + \phi_E\right)\right]$$

This is the iterative solver in general. If we designate the current iteration as m & the new iteration as m+1,

J-R method is:

$$\phi_P^{m+1} = \frac{1}{(\quad)}\left[S_P^m + \frac{D}{\Delta x^2}\left(\phi_W^m + \phi_E^m\right)\right]$$

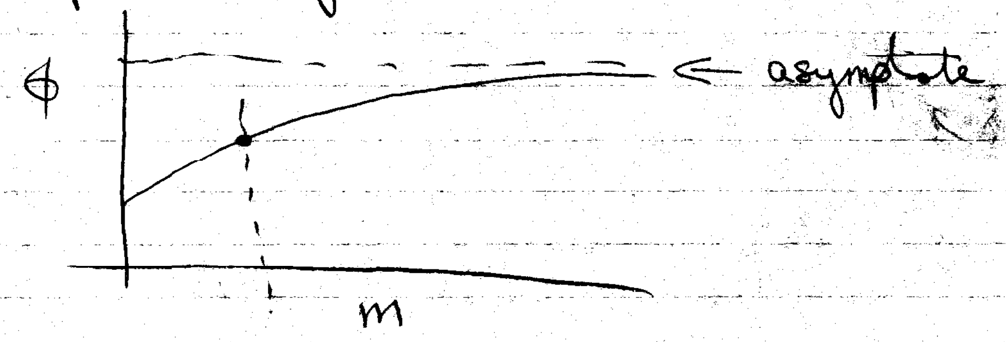
ie use only old values to update ϕ.

G-S method is:

$$\phi_P^{m+1} = \frac{1}{(\quad)}\left[S_P^m + \frac{D}{\Delta x^2}\left(\phi_W^{m+1} + \phi_E^m\right)\right]$$

ie use the latest value of ϕ available. GS does not require an inversion since West point was just updated (assuming a West to east sweep).

(b) SOR is just an extrapolated G-S to speed up convergence (J-R + G-S are slow to converge)

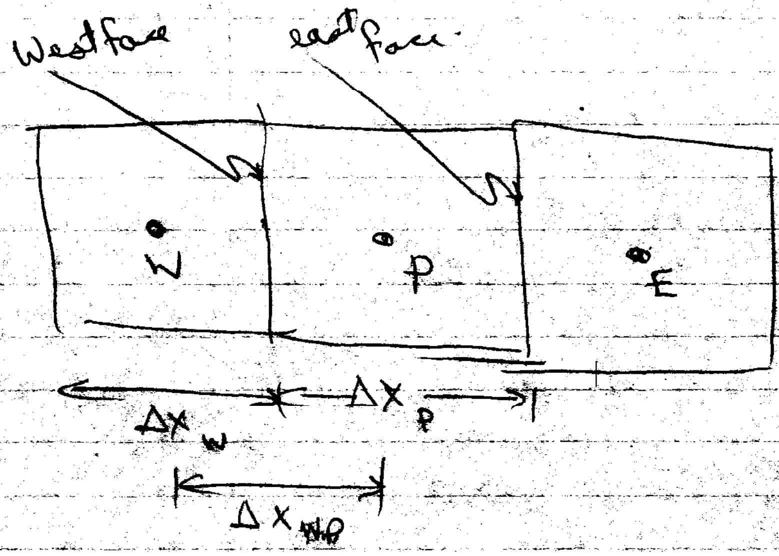


Calc. intermediate ϕ_p^* using G-S as before.

Set $\phi_p^{m+1} = \omega\, \phi_p^* + (1-\omega)\, \phi_p^m$

Where $\omega \in (1 \to 2)$.

(c) $\nabla \cdot D \nabla \phi$ in non-uniform case. assume 1-D to illustrate. $\Rightarrow \dfrac{\partial}{\partial x} D(x) \dfrac{\partial \phi}{\partial x}$



Treat the term as the slope of slopes.

$$\frac{\partial}{\partial x} D(x) \frac{\partial \phi}{\partial x} \rightarrow \frac{D_{WF}\left(\frac{\phi_W - \phi_P}{\Delta X_{WP}}\right) - D_{EF}\left(\frac{\phi_P - \phi_E}{\Delta X_{EP}}\right)}{\Delta X_P}$$

$$= \frac{D_{WF}}{\Delta X_{WP} \Delta X_P} \phi_W - \left(\frac{D_{WF}}{\Delta X_{WP} \Delta X_P} + \frac{D_{EF}}{\Delta X_{EP} \Delta X_P}\right) \phi_P$$

$$+ \frac{D_{EF}}{\Delta X_{EP} \Delta X_P} \phi_E$$

Where $D_{WF} = \frac{1}{2}(D_W + D_P)$ etc.

(d) Numerical criticality:

$$\frac{1}{v} \cancel{\frac{\partial \phi}{\partial t}}_{0} = \nabla \cdot D \nabla \phi + \left(\frac{\nu \Sigma_F}{K} - \Sigma_a\right) \phi$$

introduce the fudge factor K.

In G-S solver add an outer loop to converge on K as per

$$K^{new} = K_{old} \times \frac{\int \text{Sources current iteration} = \underline{\underline{F}}\,\phi}{\int \text{Sources past iteration} = \underline{\underline{M}}\,\phi}$$

Physically, this makes sense since if source terms are ↑ as iteration proceeds, then the fuel is too reactive, ie supercritical.
∴ Artificially suppress it by making K bigger.

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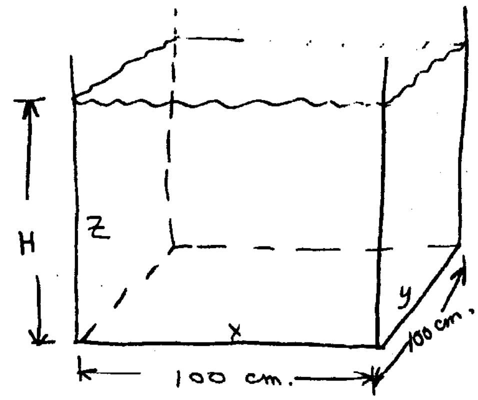
(a) $\Sigma_a = 0.500\ cm^{-1}$

$D = 10\ cm.$



$$\frac{1}{v}\frac{\partial\phi}{\partial t} = D\nabla^2\phi + (\nu\Sigma_f - \Sigma_a)\phi$$

S.S: $D\nabla^2\phi + (\nu\Sigma_f - \Sigma_a)\phi = 0$

$$\phi = \phi_0 \cos\alpha x \ \cos\beta y \ \cos\gamma z$$

$$\Downarrow$$

$$-D(\alpha^2 + \beta^2 + \gamma^2) + (\nu\Sigma_f - \Sigma_a) = 0 \Leftarrow \text{Criticality Condition}$$

& Boundary Conditions $\Rightarrow$ $\alpha = \frac{\pi}{100}$, $\beta = \frac{\pi}{100}$, $\gamma = \frac{\pi}{H}$

$$\therefore \nu\Sigma_f = \Sigma_a + D\left(\frac{3\pi^2}{(100)^2}\right) = 0.500 + 0.0296 = \boxed{.5296\ cm^{-1}}$$

(a) $= \nu\Sigma_f$

b) With absorber (which doesn't displace volume)

$$-D\left(\left(\frac{\pi}{100}\right)^2 + \left(\frac{\pi}{100}\right)^2 + \left(\frac{\pi}{110}\right)^2\right) + \nu\Sigma_f - \Sigma_a - \Sigma_{abs} = 0$$

↑ additional Σ_a due to absorber

$$\therefore \Sigma_{abs} = -10\pi^2\left[\frac{2}{100^2} + \frac{1}{110^2}\right] + 0.0296$$

$$= -0.0279 + 0.0296$$

$$\boxed{\Sigma_{abs} = 0.0017\ cm^{-1}}$$

8. (a) $$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\frac{\partial^2 \phi}{\partial x^2} + (\nu\Sigma_f - \Sigma_a)\phi$$

B.C.: $\phi(\pm a/2, t) = 0$ or $\left.\frac{\partial \phi}{\partial x}\right|_{x=0} = 0$ or same combination

I.C.: $\phi(x, 0) =$ given

(b) Explicit

$$\frac{1}{v}\frac{\phi_p^{t+\Delta t} - \phi_p^t}{\Delta t} = \frac{D}{\Delta x^2}\left(\phi_W^t - 2\phi_p^t + \phi_E^t\right) + (\nu\Sigma_f - \Sigma_a)\phi_p^t$$

$$\therefore \phi_p^{t+\Delta t} = \frac{v\Delta t D}{\Delta x^2}\left(\phi_W^t + \phi_E^t\right) + \underbrace{\left(1 - v\Delta t\left(\frac{2D}{\Delta x^2} - \nu\Sigma_f + \Sigma_a\right)\right)}_{\text{gives instability if } < 0}\phi_p^t$$

$\therefore$ for stability: $$\Delta t \le \frac{\Delta x^2}{v(2D - \nu\Sigma_f\Delta x^2 + \Sigma_a\Delta x^2)}$$

(c) For implicit:

$$\phi_p^{t+\Delta t} = \frac{v\Delta t D}{\Delta x^2}\left(\phi_W^{t+\Delta t} + \phi_E^{t+\Delta t}\right) + \phi_p^t + \left(-v\Delta t\left(\frac{2D}{\Delta x^2} - \nu\Sigma_f + \Sigma_a\right)\right) \times \phi_p^{t+\Delta t}$$

$$\therefore \phi_p^{t+\Delta t} = \frac{\frac{v\Delta t D}{\Delta x^2}\left(\phi_W^{t+\Delta t} + \phi_E^{t+\Delta t}\right) + \phi_p^t}{1 + v\Delta t\left(2D/\Delta x^2 - \nu\Sigma_f + \Sigma_a\right)}$$

Dominant term.

This will be well behaved even for large Δt. Watch out for positive reactivity, though, where $\nu\Sigma_f$ is big.

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2-group : $\chi_1 = 1$ $\Sigma_{s_{21}} = 0$ (no upscatter)
$\chi_2 = 0$ Homogeneous. $\Sigma_{f_1} \simeq 0$

$\therefore$ in steady state:

$$-D_1 \frac{\partial^2 \phi_1}{\partial x^2} + \Sigma_{R_1} \phi_1 = \nu_2 \Sigma_{f_2} \phi_2$$

$$-D_2 \frac{\partial^2 \phi_2}{\partial x^2} + \Sigma_{a_2} \phi_2 = \Sigma_{s_{12}} \phi_1$$

For bare reactor, ϕ_1 & ϕ_2 have same shape (cosine)

Usual B.C.: $\phi(\pm a/2) = 0$ (at extrapolated distance)

Let $\phi_1(x) = \phi_1 \psi(x)$, $\phi_2(x) = \phi_2 \psi(x)$

& $\nabla^2 \psi + B^2 \psi = 0 \Rightarrow \psi \sim \cos(\pi/a \, x) \Rightarrow B^2 = (\pi/a)^2$

$$\therefore \begin{bmatrix} D_1 B^2 + \Sigma_{R_1} & -\nu_2 \Sigma_{f_2} \\ -\Sigma_{s_{12}} & D_2 B^2 + \Sigma_{a_2} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = 0$$

ie $\underline{\underline{A}} \, \underline{\phi} = 0$

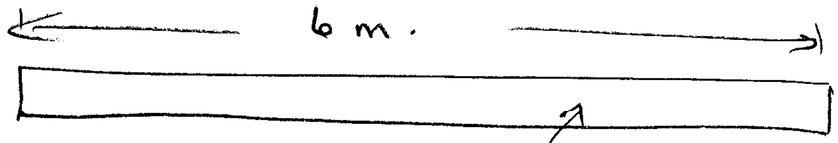
Which only has a non-trivial solution if $|\underline{\underline{A}}| = 0$

$$\therefore \boxed{(D_1 B^2 + \Sigma_{R_1})(D_2 B^2 + \Sigma_{a_2}) - \nu_2 \Sigma_{f_2} \Sigma_{s_{12}} = 0}$$

criticality condition

Where $B^2 = (\pi/a)^2$

10.



$C_p = 221\ \mathrm{J/kgK}$.

6.5 MW
12 bundles @ 20 kg each.

Time to melt (assume $\Delta T \sim 1000°c$)

$$\rho C_p \frac{\partial T}{\partial t} = \dot{q}''' \Rightarrow \underbrace{\mathrm{Vol}\cdot\rho}_{\text{Mass}} C_p \frac{\partial T}{\partial t} = \underbrace{\mathrm{Vol}\cdot \dot{q}'''}_{\text{Total power}}$$

$$\frac{\cancel{kg}}{m^3}\cdot\frac{J}{\cancel{kg}\cancel{K}}\cdot\frac{\cancel{K}}{s} = J/s.m^3$$

$$\therefore\ 20(kg)\cdot 221\left(\frac{J}{kg\,K}\right)\cdot\frac{\overset{1000°}{\Delta T\,(K)}}{\Delta t\,(s)} = \frac{6.5\times10^6}{12}\ J/s$$

$$\therefore\ \Delta t = \frac{20\times 221\times 10^3\times 12}{6.5\times10^6}\ s.$$

$$= \underline{8.16\ \text{seconds}}$$