ENGINEERING PHYSICS 4D3/6D3

DAY CLASS — Dr. Wm. Garland

DURATION: 3 hours —

McMASTER UNIVERSITY FINAL EXAMINATION — December 1999

Special Instructions:

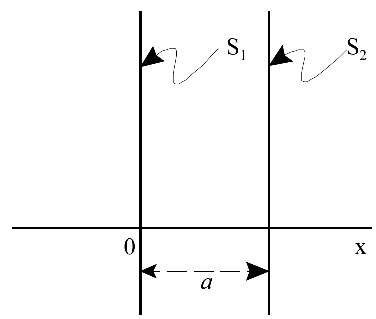
1. Closed Book. All calculators and up to 8 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions.
3. The value of each question is as indicated.
4. Point form is sufficient for discussion type questions.

   TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 3 PAGES AND 8 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [10 marks] In nuclear reactors a radioactive isotope A is created at a constant rate, R [# / s] by neutron absorption. It can subsequently undergo a decay or absorb another neutron to be transformed into another isotope B.
   a. What is the rate equation governing the concentration of A, denoted N_A? The decay constant of A is λ, the flux level φ is constant and the microscopic absorption cross section of A is σ_A.
   b. Solve the rate equation given the initial concentration of A, $N_A(0)$.

2. [10 marks] Consider two infinite planar sources of one-speed neutrons located a distance *a* apart in an infinite non-multiplying medium (scattering and absorption only) as shown in the figure.
   a. Determine the neutron flux as a function of position.
   b. Determine the neutron current as a function of position.



3. [10 marks] An infinite slab reactor, thickness *a*, is critical.
   a. What happens if the slab is divided into 2 halves (each of thickness *a*/2) and separated to open up a gap, *b*, between the two halves? Assume a vacuum in the gap. Specifically, what is the new flux distribution and are the halves critical, sub-critical or super-critical?
   b. Repeat (a) but this time for a finite slab(ie, thickness *a*, height *h*, depth *d*).

4. [10 marks] For an infinite slab reactor of extrapolated thickness *a*, derive the criticality condition for the two-group approximation (fast and thermal neutrons). Assume the slab is surrounded by a vacuum on both sides and that the slab is a homogeneous mixture of fuel and moderator. Make no simplifying assumptions about up-scatter or fissioning but discuss which variables (and hence, terms) are likely to be small enough to ignore, given typical errors in experimentally determined parameters.

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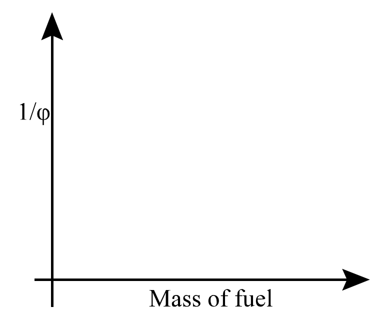
5. [20 marks] The McMaster Nuclear Reactor contains 5 control rods (normally kept at a fixed position to provide rough reactivity control) and 1 regulating rod that moves continuously to provide fine reactivity control. Both the control rods and the regulating rod must be calibrated to ensure that their reactivity worths are within specification. The 5 control rods are typically worth about 85 mk total and safety regulations require that the regulating rod be worth between 4 to 6 mk. All rods are 60 cm in length.
   a. Why should the regulating rod be worth at least 4 mk but not more than 6 mk?
   b. How can the regulating rod be calibrated? We need to know the worth (in mk) of each cm of the rod. Hint: Consider using the inhour equation:
   $$\rho = \frac{\omega \ell}{(1+\omega \ell)} + \frac{1}{(1+\omega \ell)} \sum_{i=1}^{N} \frac{\omega \beta_i}{(\omega + \lambda_i)}$$
   c. How could the calibrated regulating rod be used to calibrate the 5 control rods?
   d. Every so often, the core has to be refuelled or reorganized to accommodate new experiments. If the control and regulating rods are not repositioned to new locations in the core, would they need to be re-calibrated? If so, why? If not, why not?
   e. Where would be the best locations for the rods? Why?
   f. The flux needs to be measured to provide a signal to the regulating rod controller. Where would be the best location(s) for the flux measurement, at the regulating rod position or some distance from it? Why?

6. [10 marks] Consider a reactor like the McMaster Nuclear Reactor being brought to the critical state by the gradual addition of fuel assemblies. The reactor is initially without fuel. A neutron source, S neutrons / cm^3 - s, has been placed in the core and the assemblies are added one by one. After each assembly has been added, the flux, φ is measured, the flux is allowed to settled down to a steady state and a data point is added to a plot of $1/\varphi$ vs mass of fuel (or G_f if you prefer).



   a. What is the transient neutron (flux) balance equation? Assume one-speed neutrons, the reactor is homogeneous, the diffusion term $DL^2\varphi$ term can be replaced by a buckling term $-B^2\varphi$, and the source, S, is distributed uniformly throughout. Ignore the delayed precursors, poison and fuel depletion. Split the absorption term up into a fuel part and a non-fuel part to make the effect of the amount of fuel more explicit.
   b. Solve for the steady state flux as a function of the amount of fuel. Sketch what the above plot will look like as criticality is approached.
   c. As the plot unfolds, the critical mass can be predicted by extrapolating the $1/\varphi$ plot to the x-axis. Explain why this is so. Will this be a conservative estimate or not? Why or why not?

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7. [10 marks] Using the point kinetics approximation (no space dependence, one-speed neutrons) with one equivalent group of delayed neutrons, derive an expression for $\rho(t)$ such that $n(t) = n_0 + a\,t$ where a is some constant and n_0 is the neutron density at t=0. Hint: Use the Inverse Method. Recall:

$$\rho(t) = \beta + \frac{\Lambda}{n(t)}\frac{dn}{dt} - \beta\int_0^\infty \frac{D(\tau)n(t-\tau)}{n(t)}d\tau \quad \text{where } D(\tau) = \sum_i \frac{\lambda_i\beta_i}{\beta}e^{-\lambda_i\tau}$$

and that $\int_0^\infty te^{-\lambda t}dt = 1/\lambda^2$.

8. [20 marks] The general multigroup neutron diffusion equations with delayed precursors are given by:

$$\frac{1}{v_g}\frac{\partial\varphi_g}{\partial t} = \nabla\cdot D_g\nabla\varphi_g - \Sigma_{a_g}\varphi_g - \Sigma_{s_g}\varphi_g + \sum_{g'=1}^{G}\Sigma_{s_{g'g}}\varphi_{g'}$$
$$+ \chi_g(1-\beta)\sum_{g'=1}^{G}\nu_{g'}\Sigma_{f_{g'}}\varphi_{g'} + \chi_g^C\sum_{i=1}^{N}\lambda_i C_i + S_g^{ext}$$

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i\sum_{g'=1}^{G}\nu_{g'}\Sigma_{f_{g'}}\varphi_{g'}$$

Note that φ_g and C_i are functions of $\underline{r}$ and t but the notation has been dropped for clarity. The poison equations are:

$$\frac{\partial I(\underline{r},t)}{\partial t} = \gamma_I\sum_{g'=1}^{G}\Sigma_{f_{g'}}\varphi_{g'}(\underline{r},t) - \lambda_I\,I(\underline{r},t)$$

$$\frac{\partial X(\underline{r},t)}{\partial t} = \gamma_X\sum_{g'=1}^{G}\Sigma_{f_{g'}}\varphi_{g'}(\underline{r},t) + \lambda_I\,I(\underline{r},t) - \lambda_X\,X(\underline{r},t) - \sum_{g'=1}^{G}\sigma^X_{a_{g'}}\varphi_{g'}(\underline{r},t)\;X(r,t)$$

and the fuel depletion equation is:

$$\frac{\partial N_f}{\partial t} = -N_f(\underline{r},t)\sum_{g'=1}^{G}\sigma^f_{a_{g'}}\varphi_{g'}(\underline{r},t)$$

a. Assuming two neutron groups (fast and thermal), no upscatter, no fast fissions, and no neutrons born in the thermal region, what are the steady state flux and precursor equations? Show that the delayed precursors do not appear in the flux equations. Justify this physically.

b. In a real reactor, a true steady state could never be achieved. Why? Explain your answer by considering the control rod position as a function of time seconds, minutes, hours, days ... after criticality is first reached.

c. Consider a simple one dimensional homogeneous slab reactor. Outline briefly a numerical procedure to calculate the so-called steady state flux (plus any other parameters as needed) over a period of seconds, minutes, hours, ... Use the fudge factor k to keep the reactor critical over time.

d. Relate how k in (c) varies to how the control rod position in (b) varies.