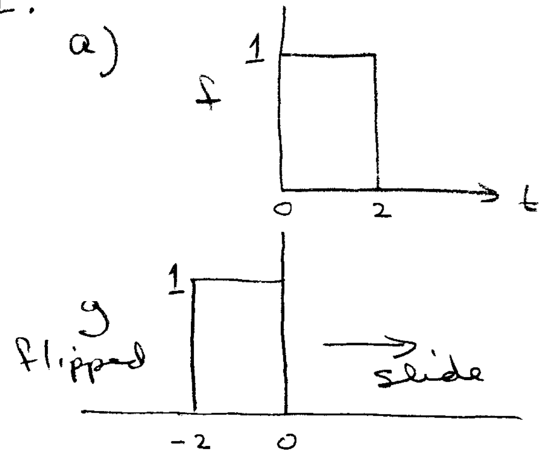
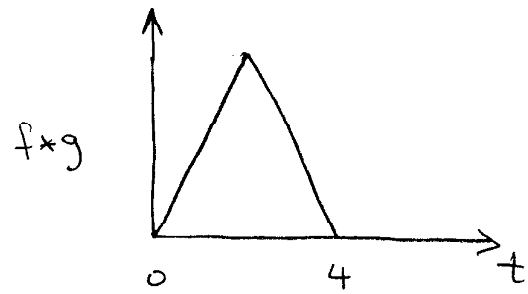
Solutions

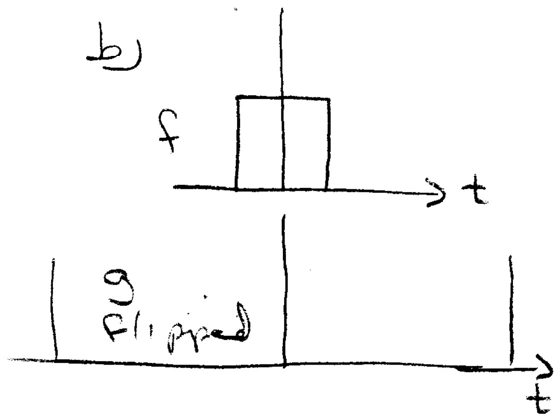
1.

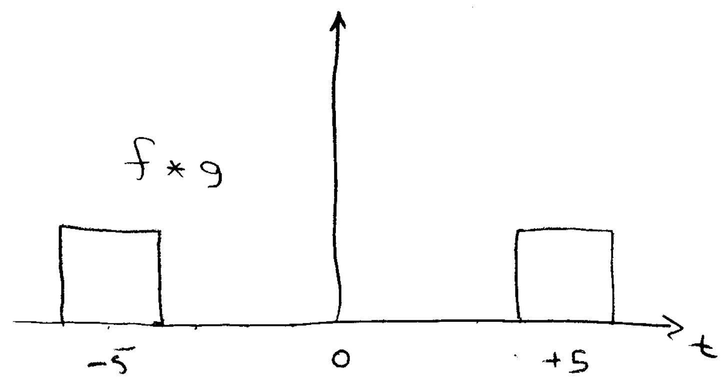
a)



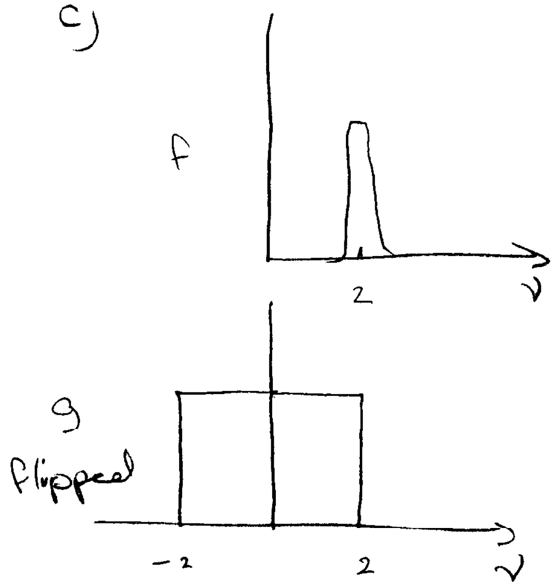


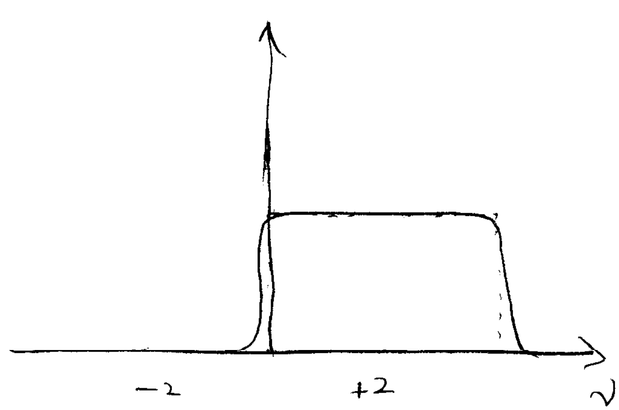
b)





c)



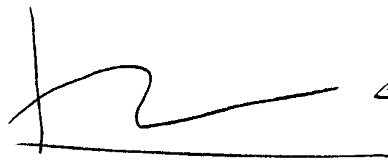


2. (a) 1. $\int_{-\infty}^{\infty} |f(t)|^2 \, dt < \infty$

This implies $f(t) \to 0$ as $t \to \infty$

2. $f(t)$ & FT of $f(t)$ are single valued

ie



← not allowed.

3. $f(t)$ & FT of $f(t)$ are piecewise continuous.

eg →

4. $f(t)$ & FT of $f(t)$ are bounded (all real signals satisfy this)

(b) $f(t) = E(t) + O(t)$

$\therefore f(-t) = E(-t) + O(-t) = E(t) - O(t)$

$\therefore f(t) + f(-t) = 2 \cdot E(t)$

& $f(t) - f(-t) = 2 \cdot O(t)$

QED

2. (c) FT of $O(t)$, an odd function,

$$= \int_{-\infty}^{\infty} O(t)\, e^{-2\pi i \nu t} dt$$

$$= \int_{-\infty}^{\infty} O(t) \cos(2\pi \nu t)\, dt + i \int_{-\infty}^{\infty} O(t) \sin(2\pi \nu t)\, dt$$

$\|$ zero since cos is even & $O(t)$ is odd $\therefore \int \Rightarrow 0$

$\| \; 2i \int_0^{\infty} O(t) \sin(2\pi \nu t)\, dt$ since both $O(t)$ & sin are odd.

$\therefore$ FT of $O(t)$ is purely imaginary.

3. (a) $f(t+a) \rightleftharpoons F(\nu) e^{2\pi i \nu a}$

(b) $\Pi_a(t) \rightleftharpoons a \operatorname{sinc}(\pi \nu a)$

(c) $e^{-\pi t^2} \rightleftharpoons e^{-\pi \nu^2}$

(d) $\delta(t) \rightleftharpoons 1$

(e) $\delta(t-a) \rightleftharpoons e^{-2\pi i \nu a}$

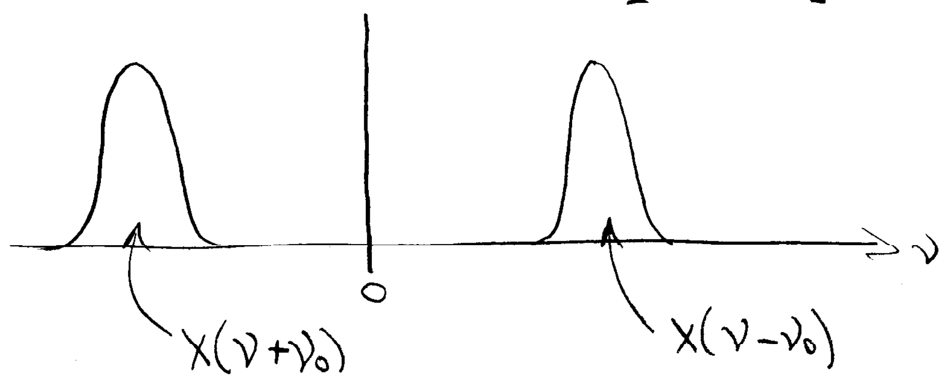
4. (a) If $\delta(t-t_0)+\delta(t+t_0) \rightleftharpoons 2\cos(2\pi \nu t)$

then $\cos 2\pi\nu_0 t \rightleftharpoons \frac{1}{2}\left[\delta(\nu-\nu_0)+\delta(\nu+\nu_0)\right]$

Since the functions are real & even.

You have proved this in the assignment on chapter 5.

(b) $x(t)\cos(2\pi\nu_0 t) \rightleftharpoons \frac{1}{2} X(\nu) * \left[\delta(\nu-\nu_0)+\delta(\nu+\nu_0)\right]$



(c) Notice how the original signal ($x(t)$ or $X(\nu)$) is shifted in the freq. domain. By choosing a different carrier freq., ν_0, by setting the 'dial' on a radio, we can broadcast and recieve the same information on different frequencies.

5\. (a)

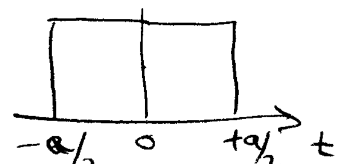
In general $\frac{d}{dt}[f(t) * g(t)] = \frac{df}{dt} * g = f * \frac{dg}{dt}$

Let $f = e^{-\pi t^2}$ & $g = \Pi_a(t)$

We could use $\frac{df}{dt} * g$ but that turns out to be messy.

Try $f * \frac{dg}{dt} = e^{-\pi t^2} * \frac{d}{dt}\Pi_a(t)$

Now $\Pi_a(t) =$



By inspection, the slopes are 0 except at $\pm a/2$.

Thus $\frac{d\,\Pi_a(t)}{dt} = \delta(t + a/2) - \delta(t - a/2)$

$\therefore f * \frac{dg}{dt} = e^{-\pi t^2} * [\delta(t + a/2) - \delta(t - a/2)]$

$= e^{-\pi \nu^2} \cdot 2i \sin(2\pi\nu\, a/2)$

$= e^{-\pi \nu^2} \cdot 2i \sin(\pi \nu a)$

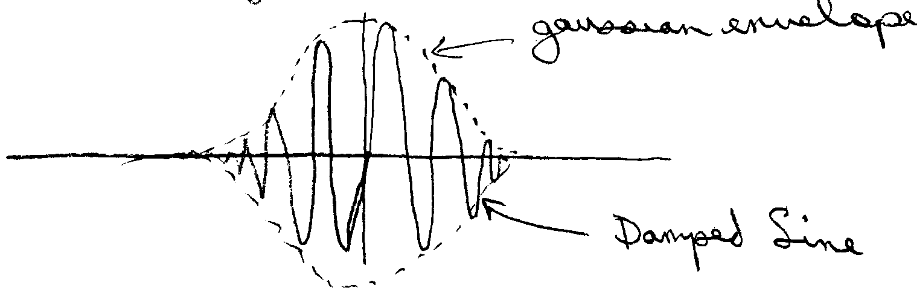
Alternatively, use $\frac{d}{dt} f(t) \rightleftharpoons 2\pi i \nu F(\nu)$

to directly get:

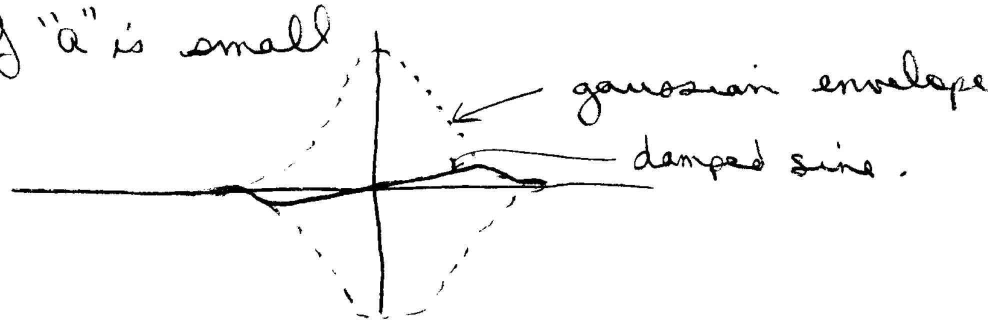
$\frac{d}{dt}[e^{-\pi t^2} * \Pi_a(t)] \rightleftharpoons 2\pi i \nu\, e^{-\pi\nu^2}\, a\, \mathrm{Sinc}(\pi \nu a)$

$\qquad\qquad = 2i\, e^{-\pi\nu^2} \sin \pi \nu a$

(b) If "a" is large:



If "a" is small



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