

1. [20 marks]

(a)

$$\int_{-\infty}^{+\infty} \mathbf{d}(-3t)(1 + \cos^2 pt) dt = \frac{1}{|-3|} (1 + \cos^2 \frac{p \cdot 0}{-3}) = \frac{2}{3}$$

[5 marks]

(b)

$$\int_{-\infty}^{+\infty} \mathbf{d}(t - \frac{1}{2}) \frac{t^2}{(t^2 + 7)} dt = \frac{(\frac{1}{2})^2}{(\frac{1}{2})^2 + 7} = \frac{1}{29}$$

[5 marks]

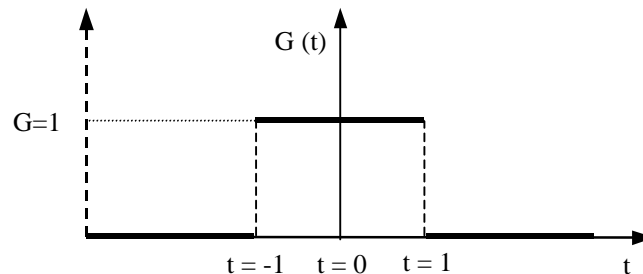
(c)

$$\int_4^7 \mathbf{d}(t - 2) f(t) dt = 0$$

(since, if treated as an ordinary function, $\delta(t-2) = 0$ for $t \in [4,7]$)

[5 marks]

(d)



[5 marks]

2. [20 marks]

(a) If $y(x)$ is linear then $y(0) = y(0+0) = y(0) + y(0)$ and therefore $y(0) = 0$. Take $y(x) = ax + b$. It follows that $y(0) = a \cdot 0 + b = b$, and therefore $ax + b$ is linear if and only if $b = 0$.

[10 marks]

(b)

$$y(t) = \int_{-\infty}^{+\infty} x(t) h(t - \mathbf{t}) dt = \int_{-\infty}^{+\infty} x(t) \mathbf{d}(t - \mathbf{t}) dt = x(t)$$

Physically, this means that, if the system response to an impulse is an impulse, then what goes in, goes out, i.e. $y(t) = x(t)$.

[10 marks]

3. [20 marks]

The output signal can be presented as

$$y(t) = \int_{-\infty}^{+\infty} x(\mathbf{t})h(t - \mathbf{t})d\mathbf{t} = x * h$$

When

$$x(t) = e^{2\pi i v_0 t} ,$$

(then)

$$\begin{aligned} y(t) &= \int_{-\infty}^{+\infty} e^{2\pi i n_0 t} \cdot h(t - \mathbf{t})d\mathbf{t} = \int_{-\infty}^{+\infty} e^{2\pi i n_0 (t - \mathbf{t}')} \cdot h(\mathbf{t}')d\mathbf{t}' \\ &= e^{2\pi i n_0 t} \cdot H(\mathbf{n}_0) \end{aligned}$$

where $H(v_0)$ is the F.T. of $h(t)$. The system response to an exponential function is another exponential function (modified by an amplitude $H(v_0)$).

[20 marks]

Total number of points: 60

Date: Jan 25, 2000