Engineering Physics 3W4

"Acquisition and Analysis of Experimental Information"

Part II: Fourier Transforms

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Chapter 1 Introduction

1.1 What this course is about and how we will proceed

This course is about Fourier Transforms, what they are, why they are useful and how to use them. Any topic should be studied in a context for best effect. Consequently, we set the scene for FT by first looking at linear systems to show why FTs are useful.

Linear systems give rise to the convolution integral. To aid in the solution of this integral, it is advantageous to transform it into a more manageable form. This is where FTs come in. It turns out that an FT is much more than a mathematical aid. It is an understanding aid; the transformation in coordinate space provides a fresh and illuminating view of the original data.

To provide a better understanding of this tool, the FT is developed from the Fourier Series which the reader may be more familiar with. Complex number notation will be necessary so we will have to make a side trip there to ensure full understanding.

We can then proceed to the FT and study it in some details, exploring its properties and physical interpretation. This will occupy us for the bulk of the course.

Our end goal is the application of FTs to practical signal analysis situations. We will be able to study only a few applications within this course but the hope is that a firm understanding of FTs that should be acquired in the study of the material herein will enable the reader to apply FTs to the various situations that will undoubtedly arise in future work.

Figure 1 summarizes the above rationale.

1.2 Learning Objectives / Outcomes

Here are some things that you should know how to do by the end of this course.

Chapter 1

1.2.1 Draw a concept map for the course showing the interrelationships between the various concepts developed in the course. (This would be a good review exercise at the end of the course but it is useful to incrementally do this as you proceed through the course.)

Chapter 2

1.2.2 Define linear systems and give examples of linear and non-linear systems.

1.2.3 Define convolution and show how it arises in linear systems.

1.2.4 Define the Fourier Transform pair and discuss why it is useful in linear systems.

Chapter 3

1.2.5 Calculate the coefficients for the Fourier Series given a particular signal.

Chapter 4

1.2.6 Manipulate expressions involving complex notation.

1.2.7 Describe complex numbers using various notations.

Chapter 5

1.2.8 Derive the Fourier Transform pair with help.

1.2.9 Predict when the FT will be real or imaginary.

1.2.10 Describe why we reflect the time signal about the origin before we take the FT.

1.2.11 Calculate the FT for special cases.

1.2.12 Recall the FT for special cases.

Chapter 6

1.2.13 Manipulate FT and Inverse FT expressions.

1.2.14 Manipulate expressions involving the convolution.

1.2.15 Visualize / sketch f*g given f and g.

1.2.16 Define and discuss autocorrelation.

1.2.17 Define all theorems.

1.2.18 Discuss aliasing, define the sampling rate criteria.

Chapter 7

1.2.19 Derive the equation for diffraction and calculate a pattern given an aperture

Chapter 8.

1.2.20 Define common filters and discuss how they work.

Chapter 9

1.2.21 Define common windows and discuss how they work.

1.2.22 Calculate sampling limits for DFT.

1.3 Reference Material

I was unable to find a single text that contained all the material covered in this course. I settled on preparing these course notes based on the sequence followed by James. James, however, is light on explanatory derivations that I feel are essential for proper learning. Thus, my course notes augment James by pulling from various sources, notably Peebles, Johnson, Fante ... The following is a list of selected texts and a brief commentary on them.

1.3.1 R. N. Bracewell, The Fourier Transform and its Applications, McGraw-Hill Book Company, 2^{nd} edition 1978, ISBN 0-07-000013-x, library reference QA 403.5.B7. *Very comprehensive but a bit heavy on the math to the point that it is hard to see the forest for the trees. A great reference text once you are underway in the field. Nice treatment of convolutions in a graphical sense, in fact, Bracewell is big on the graphical interpretation of things..*

1.3.2 J. S. Broch, Principles of Experimental Frequency Analysis, Elsevier Science Publishing, 1990, ISBN 1-85166-554-4, library reference TK 5102.5.B694. *I liked this book and considered using it as the course text. It has a nice practical ring to it but there are serious gaps in theorems and such.*

1.3.3 R.L. Fante, Signal Analysis and Estimation, An Introduction, John Wiley & Sons, 1988, ISBN 0-471-62425-X, library reference TK 5102.5.F36. *Fante has a nice treatment of digital sampling and aliasing, which I follow for Section 6.5.4 and Chapter 9 of my course notes.*

1.3.4 J. F. James, A Student's Guide to Fourier Transforms, Cambridge University Press, 1995, ISBN 0 521 46829 9. *An inexpensive, small, readable text that makes for a good introduction and guide to practical Fourier Transforms. This text forms the basis of the course notes.*

1.3.5 D. E. Johnson et al, Basic Electric Circuit Analysis, Prentice-Hall, 1984, ISBN 0-13-060111-X. *This has a nice treatment of complex numbers and I use it for Chapter 4 of the course notes*.

1.3.6 A. Papoulis, Signal Analysis, McGraw-Hill, 1977, ISBN, 0-07-048460-0. *Nice treatment, right up there with Fante. Worth a look for sure.*

1.3.7 P. Z. Peebles, Probability, Random Variables, and Random Signal Principles, McGraw-Hill Book Company, 1993. *This is on the reading list for the term I part of this course. I like the treatment of linear systems and I use it for Chapter 2 of these course notes.*

1.3.8 M. R. Spiegel, Theory and Problems of Fourier Analysis with applications to Boundary Value Problems, Shaum's Outline Series, McGraw-Hill, 1974, Library Reference QA 403.5.S66.

1.3.9 Web links. *Check out my web page at http://epic.mcmaster.ca/~garlandw/ep3w4 for links to on-line tutorials and downloadable code relating to Fourier Transforms.*

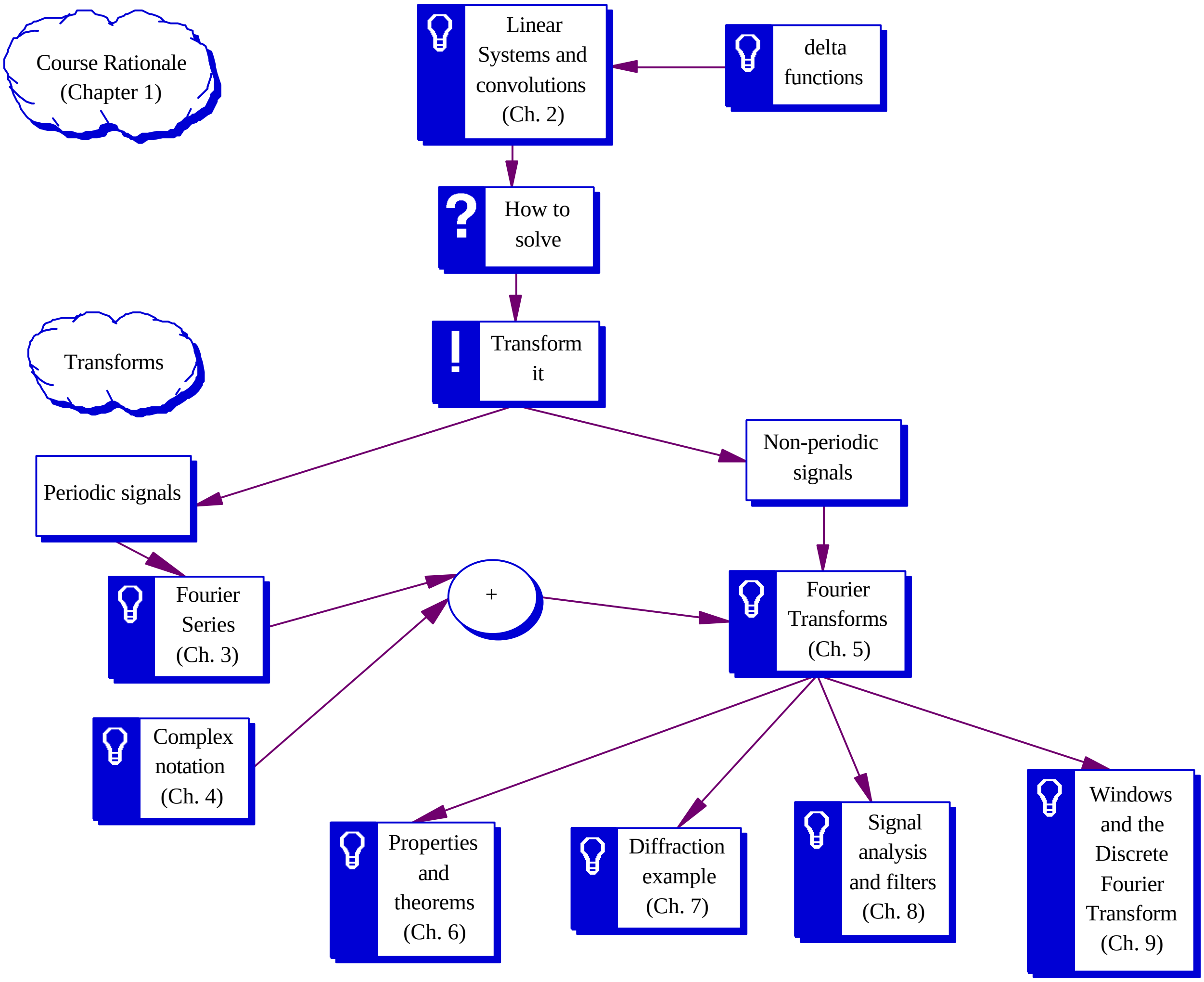

Figure 1.1 Course overview and Fourier Transform rationale