Name: A. Einstein Student ID: $E = mc^2$.

**EP 2H04 / Phy 2H04**

DAY CLASS Dr. Wm. Garland
DURATION: 50 minutes
McMASTER UNIVERSITY TEST # 1 February 15, 2001

**Special Instructions:**

1. Closed Book. All calculators and up to 4 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions. Place your answers on the exam sheets; use additional pages if necessary.
3. The value of each part is as indicated. TOTAL Value: 100 marks

**THIS EXAMINATION PAPER INCLUDES 4 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.**

1. [20 marks total] Express mathematically the zeroth, first, second and third laws of thermodynamics and state their physical meanings in one sentence each.

(5) $0^{th}$ Law: Temperatures of bodies in equilibrium are equal, ie $T_A = T_B$ if A + B are in equilibrium.

(5) $1^{st}$ Law: $dE = dQ - dW + \mu dN$
This is conservation of energy, change in energy is that due to heat change, work done + diffusive changes

(5) $2^{nd}$ Law: $\Delta S \geq 0$, Entropy ($\equiv k \ln \Omega$) of a system increases as it approaches equilibrium. $\Delta S = 0$ at equilibrium.

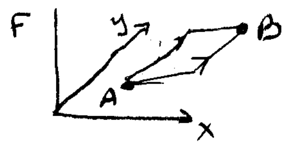
(5) $3^{rd}$ Law: $S \to 0$ as $T \to 0$. Only 1 state at $T = 0$.

2. [12marks total]
   a. How would you determine if a certain differential, g(x,y) dx + h(x,y) dy, is exact?

(3) $dF = g\,dx + h\,dy$ is exact if $\dfrac{\partial g}{\partial y} = \dfrac{\partial h}{\partial x}$

b. What is the physical meaning of the concept of exact and inexact differentials and give an example of each?

(3)




As you move from A to B, the value of F will change. The change is <u>independent</u> of the path taken for an <u>exact</u> differential. It is path <u>dependent</u> for an inexact diff.

Example of exact: $F = x^2 y \Rightarrow dF = 2xy\,dx + x^2\,dy$

Example of inexact: $dF = 3xy\,dx + x^2\,dy$

c. Convert $e^{25}$ to base 10.

(3)

$e^{25} = 10^y$ ; $y = 25 \ln(e)/\ln(10) = 25/2.3026 = 10.85$

$\therefore e^{25} = 10^{10.85}$

d. Convert $2^{10^{25}}$ to base 10

(3)

$2^{(10^{25})} = 10^{10^{25} \times \ln(2)/\ln(10)} = 10^{0.301 \times 10^{25}}$

3. [9 marks total]

a. For a small system of 100 experimental trials, 10 successes were found. What is the standard deviation, σ?

(3)

$p = \bar{n}/N = 10/100 = 0.1 \Rightarrow q = 0.9$

$\sigma = \sqrt{Npq} = \sqrt{100 \times 0.1 \times 0.9} = 3$

$\underline{\underline{\sigma = 3}}$

b. What is the relative standard deviation, $\sigma/\bar{n}$ ?

(3)

$\frac{\sigma}{\bar{n}} = \frac{3}{10} = \underline{\underline{0.3}}$

c. What would you expect to happen to σ and $\sigma/\bar{n}$ as more experiments are performed?

(3)

σ would increase $\propto \sqrt{N}$

$\frac{\sigma}{\bar{n}}$ would decrease $\propto \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$



But it also would be refined as $\bar{n}$ estimate improves.

4. [12 marks total]

a. Starting with the standard definition of work, show that P dV is a work term.

(3) $dW = F \cdot ds = P \cdot \underbrace{A \cdot ds}_{dV} = P \cdot dV$

b. How is the quantity "entropy" defined? What is its physical significance?

(3) $S \equiv k \ln \Omega$. It is a measure of the number of states of a system.

c. State the Fundamental Postulate.

(3) An isolated system in equilibrium is equally likely to be in any of its accessible states. $\left(P_i = \frac{\Omega_i}{\Omega_0}\right)$

d. State the Equipartition Theorem.

(3) On average, the internal energy is distributed equally among the degrees of freedom whose energies are expressible in the form $bq^2$

5. [20 marks total]

a. What is the probability, $P_1$, of being in any particular state, given $\Omega_0$, the number of states in a system?

(5) $P_1 = \frac{1}{\Omega_0}$

b. What is the probability, $P_i$, of being in any particular sub-set of states, given $\Omega_0$, the number of states in a system and $\Omega_i$, the number of states in a sub-system?

(5) $P_i = \frac{\Omega_i}{\Omega_0}$

c. How is $\Omega_0$, the number of states in a system, related to energy, E, typically?

(5) $\Omega_0 = \text{const } E^{\mathcal{R}/2}$, $\mathcal{R} = \text{total \# of d.o.f.} = \nu N$

where $\nu$ = # of dof per particle, $N$ = # of particles

d. Illustrate that the number of available states increases dramatically for even a small increase in energy in a system.

(5) Let $E_2 = 1.001 E_1$

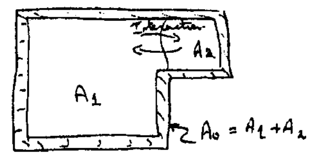
$\therefore \frac{\Omega_2}{\Omega_1} = \left(\frac{1.001 E_1}{E_1}\right)^{\mathcal{R}/2} = (1.001)^{\mathcal{R}/2}$



$\mathcal{R}$ is typically $\sim 10^{28}$ $\therefore \frac{\Omega_2}{\Omega_1} = (1.001)^{10^{28}}$

Which is a huge increase.

6. [15 marks total] Recalling the system defined by the figure, as discussed in class, we found that the total number of available states as a function of the energy partition, $E_1$, <u>peaked</u> at a particular energy. Yet we also know that the number of available states of a system, as a function of energy, increases as E increases, with <u>no peak</u>. How can both these facts be true?

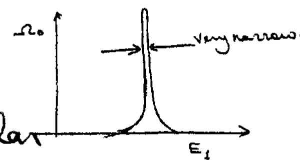



$E_0 = E_1 + E_2 = \text{constant}$

$\Omega_0 = \Omega_1 \Omega_2$

(15)

The peak refers to the fact that at a particular <u>partition</u> of energy (ie in this case, a particular value of $E_1 + E_2$), the number of possible states is maximized. The total energy, $E_0$, is held constant in that exercise.




If $E_0$ were to increase, many more states become available. For this new $E_0$, there is still a <u>partition</u> of energy that maximizes the number of states available.

7. [12 marks total]

a. Show that dS = dQ/T when V and N are held constant.

(6)

$$S(E+dQ) = S(E) + \underbrace{\left.\frac{\partial S}{\partial E}\right)_{V,N}}_{\equiv \frac{1}{T}} dQ + \cancel{\frac{1}{2}\frac{\partial^2 S}{\partial E^2}(dQ)^2} + \cdots$$

drop higher order terms

$$\therefore S(E+dQ) - S(E) = dS = \frac{dQ}{T}$$

Note: Because we hold V + N const, $dE = dQ - \cancel{dW}^{\,0} + \mu\, \cancel{dN}^{\,0}$

b. If a system with entropy, $S_1$, and number of states, $\Omega_1$, is somehow changed to $S_2$ and $\Omega_2$, what is $\Omega_2/\Omega_2$ in terms of $\Delta S = S_2 - S_2$?

(6)

$$S = k \ln \Omega \quad \therefore \quad S_2 - S_1 = k\ln\Omega_2 - k\ln\Omega_1 \equiv \Delta S$$

$$\therefore \Delta S = k(\ln\Omega_2 - \ln\Omega_1) = k \ln(\Omega_2/\Omega_1)$$

$$\therefore \frac{\Omega_2}{\Omega_1} = e^{\Delta S/k}$$

---THE END---