Microscopic $\Longleftrightarrow$ Macroscopic

Statistics

Small systems

large systems

Internal Energy

degrees of freedom

equipartition theorem

E + 1st Law

$dE = dQ - dW + \mu\, dN$

Entropy

- states of a system

$S \equiv k \ln \Omega$

$\Omega \propto E^{N/2}$

$\Delta S \geq 0$ 2nd Law

$S \to 0$ as $T \to 0$ 3rd law

Interactions

Thermal

$dQ = T\,dS$

0th Law $\frac{1}{T} \equiv \left.\frac{\partial S}{\partial E}\right)_{V,N}$

T's equal at $\equiv^m$ $c_V \equiv \left.\frac{\partial Q}{\partial T}\right)_V$

Heat Transfer

Mechanical

$dW = F\,ds$

$= P\,dV$

$\beta \equiv \frac{1}{V}\left.\frac{\partial V}{\partial T}\right)_P$ coeff of vol. expansion

$\kappa \equiv -\frac{1}{V}\left.\frac{\partial V}{\partial P}\right)_T$

Isothermal compressibility

Diffusive

$\Delta S > 0$

$\Rightarrow$ hot $\to$ cold

higher P expands

flow to lower potential

Constraints

Imposed

$dP = 0,\ dT = 0,\ dQ = 0$

Reversable

$\downarrow$

Engines

Natural

$\Downarrow$ Ideal gas + non-ideal gas

Maxwell's relations

G = Gibbs free energy

$= E - TS + PV = \mu N$

F = Helmholtz free energy

$= E - TS$

H = Enthalpy $= E + PV$

Chapter 3: Summary

Small Systems

$\bar{f} = \sum_s P_s f_s$ ← same function; P_s ↑ probability

$\overline{f+g} = \bar{f} + \bar{g}$

$\overline{cf} = c\bar{f}$

$p + q = 1$

$$(p+q)^N = \sum_{n=0}^{N} \underbrace{\frac{N!}{n!(N-n)!}}_{\text{\# of diff. configs of N elements for which n satisfy a criterion (Binomial coefficient)}} \overbrace{p^n q^{N-n}}^{\text{prob that n elements satisfy a criterion + the rest do not.}} = 1$$

$$\boxed{\begin{array}{l}\text{Prob of being in a given state:} \\ P_N(n) = \dfrac{N!}{n!(N-n)!} p^n q^{N-n} \\ \text{PN chosen n}\end{array}}$$

Stirling's formula: $\ln(m!) \simeq m \ln m - m + \frac{1}{2}\ln(2\pi m)$

Trick: $e^x = 10^y \Rightarrow x = \ln 10^y = y \underbrace{\ln 10}_{2.3026} \Rightarrow e^x = 10^{x/2.3026}$

(notes 4-10) [in general: $\underline{(b_1)^{x_1} = (b_2)^{x_2}} \Rightarrow x_2 = x_1 \ln(b_1)/\ln(b_2)$]

Chapter 4: Large Systems

$p \equiv \bar{n}/N$ (← mean), $\sigma^2 = \overline{(n-\bar{n})^2} = \sum_1 P_n (n-\bar{n})^2$

$\sigma/\bar{n} = \sqrt{q/Np}$ $\qquad = Npq = (\text{standard deviation})^2$

$$P_N(n) \simeq \frac{(\bar{n})^n}{n!} e^{-\bar{n}} \Leftarrow \text{Poisson } (p \ll 1) \leftarrow \text{rare successes}$$

$$P(n) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(n-\bar{n})^2/2\sigma^2} \Leftarrow \text{Gaussian } (n-\bar{n}) \ll \sigma^2$$

<u>N big</u> only

Random walk: $\overline{(s-\bar{s})^2} = \overline{s^2} - \bar{s}^2 \leftarrow \left[\int_{-\infty}^{\infty} s P(s) ds\right]^2$

$\equiv \overline{(\Delta s)^2}$ $\qquad \overline{s^2}$: ↑ $\int_{-\infty}^{\infty} s^2 P(s) ds$

$\bar{S} = N\bar{s}$, $\sigma^2 = N\overline{(\Delta s)^2}$ $\qquad 1\,\mathring{A} = 10^{-10}$ m.

Chapter 5 Summary

Internal Energy & Equipartition

I.E. distributed equally among dof's of energy in bq^2

k = Boltzmann's Const

$= 1.381 \times 10^{-23}$ J/K

$= 0.846 \times 10^{-4}$ eV/K

$\bar{\varepsilon} = \frac{1}{2} kT$ per dof

$$K.E. = \frac{1}{2} m v^2 = \frac{p^2}{2m} = \frac{(mv)^2}{2m}$$

$$P.E. = \frac{1}{2} k x^2 = \int_0^x \underbrace{f_{applied}}_{= +kx}\, dx$$

Chapter 6 Summary

Internal Energy & the 1st Law

$kT \approx 0.025$ eV at $T = 300$K

$dE = dQ - dW + \mu dN$ ← 1st Law

$W = \int_S \bar{F} \cdot d\bar{s}$

1 particle

$$\varepsilon_{(1\,dof)} = \mu + \underbrace{\overbrace{\tfrac{1}{2} k r^2}^{u,\ \text{potential}} + \overbrace{\tfrac{1}{2} m v^2}^{\text{kinetic}}}_{\text{thermal}}$$

N particles

$$E = E_{thermal} + \mu N$$

dE = exact ← path independent

dQ, dW = inexact ← path dependent

$$dF = \underbrace{g}_{\frac{\partial F}{\partial x}}\, dx + \underbrace{h}_{\frac{\partial F}{\partial y}}\, dy \quad \text{if} \quad \frac{\partial g}{\partial y} = \frac{\partial h}{\partial x}$$

Chapter 7 Summary

The States of a System

$P = \frac{1}{\Omega_0}$ ← # of states

↑ prob.

$$\Omega_0 = \prod_{i=1}^{R} \Omega_i \qquad R \leftarrow \text{total \# dof}$$

$P_i = \frac{\Omega_i}{\Omega_0}$ ← subsystem / ← full system

$\Omega(E, dE) = g(E)\, dE$

energy spacing $= 10^{-24}$ eV

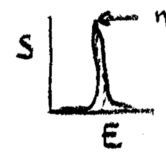
$\Omega_i(E) \propto E^{\chi}$ ← typically $\frac{1}{2}$ $\Rightarrow \Omega_0 \propto E^{R/2}$

Chapter 8 Summary
Entropy + the 2nd Law

$\Delta \Omega_0 \geq 0 \Leftarrow$ 2nd Law

$S \equiv k \ln \Omega \Rightarrow \Delta S \geq 0$
[J/K]

Examples of



Chapter 9 Summary
The Thermal Interaction

$S = k \ln \Omega = k \frac{\mathcal{R}}{2} \ln E + \ln(\text{const})$

$\Omega = \text{const}\; E^{\mathcal{R}/2}$

Since $\Delta S_0 \geq 0 \Rightarrow \frac{\Delta E_1}{T_1} + \frac{\Delta E_2}{T_2} \geq 0$
Heat flows from hot → cold.

$\left.\frac{\partial S}{\partial E}\right)_{V,N} \equiv \frac{1}{T}$, T's equal at $\equiv^m$ ← 0th Law

T = measure of how Ω varies with E.

$\left.\frac{\partial S}{\partial E}\right)_{V,N} = \frac{k\mathcal{R}}{2} \cdot \frac{1}{E} \equiv \frac{1}{T}$

$\Rightarrow E = \frac{1}{2} k \mathcal{R} T = \frac{1}{2} k \nu N T$ ($\mathcal{R} = \nu N$)

$dS = \underbrace{\left.\frac{\partial S}{\partial E}\right)_{V,N}}_{1/T} dE$ if V,N const. (dE " dQ)

$\therefore\; dQ = T\,dS$

$E_{Total} = \frac{1}{2} k \nu N T + \mu N$

Note, μ may change during phase transition | define T + generate k $\Leftarrow \varepsilon = \frac{1}{2} kT$ (per dof)

$\Delta S = k \ln \Omega_2 - k \ln \Omega_1$
$= k \ln\left(\frac{\Omega_2}{\Omega_1}\right) = \frac{\Delta Q}{T}$

$\therefore\; \frac{\Omega_2}{\Omega_1} = e^{\Delta Q/kT}$

$S_0 = k \ln \Omega_0 \Rightarrow \Omega = e^{S_0/k}$

$S_0(E_1) = S_0(\bar{E}_1 + \Delta E_1) = S_0(\bar{E}_1) + \underbrace{\frac{\partial S_0}{\partial E_1}}_{0 \text{ at } \equiv^m} \Delta E_1 + \frac{1}{2} \frac{\partial^2 S_0}{\partial E_1^2} \Delta E_1^2$
$= S_0(\bar{E}_1) - \frac{\mathcal{R} k}{4 \bar{E}_1^2} \Delta E_1^2$

$\therefore\; P(E_1) = \text{const}\; \Omega_0(E_1) = \text{const}\; e^{S_0(E_1)/k}$
$= \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\Delta E_1^2/\sigma^2} \Leftarrow$ Gaussian

$\sigma = \sqrt{\frac{2}{\mathcal{R}}}\; \bar{E}$ ($\bar{E} = \mathcal{R}\bar{\varepsilon}$)

$C_V \equiv \left.\frac{\partial Q}{\partial T}\right)_V$, $C_P \equiv \left.\frac{\partial Q}{\partial T}\right)_P$ (J/K)

(J/K) → $c_V' = \frac{1}{m} \left.\frac{\partial Q}{\partial T}\right)_V$ (m: mass)

$c_V = \frac{1}{n} \left.\frac{\partial Q}{\partial T}\right)_V$ (n: moles)

$\Delta Q = \Delta E = \frac{\nu N}{2} kT$

$\Downarrow$

$C_V = \frac{\nu}{2} N k = \frac{\nu}{2} n R$

1 calorie = 4.18 J

molecules/cc
$= \frac{\mathcal{A} \cdot \rho}{A}$ $\frac{\#/\text{gm-mole} \times \text{gm}/cc}{\text{gm}/\text{gm-mole}}$

$\mathcal{A} = 6.0221367 \times 10^{23}$

Chapter 9 - cont'd

Const V, N $\quad S(T_0) = \int_0^{T_0} dS = \int_0^{T_0} \frac{dQ}{T} = \int_0^{T_0} \frac{C_v\,dT}{T}$

$\therefore C_v \to 0$ as $T \to 0$

$S = k \ln \Omega = k \ln 1 = 0$ at $T = 0$ (1 state)

$S \to 0$ as $T \to 0$ $\Leftarrow$ 3rd Law

Heat Transfer: $\frac{\Delta Q}{\Delta t}$ = H.T. rate = $-kA\frac{dT}{dx}$ $\Rightarrow$ $T(x) = -\frac{1}{kA}Hx + T_2$

Conduction $\rightarrow$ (H) $\quad$ thermal conductivity W/m°C $\Rightarrow$ $\int dT = -\frac{1}{A}H\int \frac{dx}{k}$

$\therefore \Delta T = -\frac{H}{A}\sum_i \frac{L_i}{k_i}$

$\frac{L}{k} \equiv R$ (R-factor) $\quad \frac{ft^2\ {}^\circ F \cdot h}{BTU}$

Convection

$\frac{H}{A} = q'' = h(T - T_\infty)$

└ H.T. coeff. W/m²°C

Radiation

Power (watts) $= \sigma A e T^4$ Stefan's Law

σ = 5.6696 × 10⁻⁸ W/m² K⁴

e = 1 absorber

= 0 reflector

Chapter 10 Summary

The Mechanical Interaction

$dW = \underline{F}\cdot d\underline{s} = P\,dV$

$dE = dQ - dW + \mu\,dN \Rightarrow T = \left.\frac{\partial E}{\partial S}\right)_{V,N}, \; P = \left.\frac{\partial E}{\partial V}\right)_{S,N}, \; \mu = \left.\frac{\partial E}{\partial N}\right)_{S,V}$

$dQ = T\,dS$ since $\left.\frac{\partial S}{\partial E}\right)_{V,N} = \frac{1}{T}$

$dS = \frac{1}{T}dE + \frac{P}{T}dV - \frac{\mu}{T}dN \Rightarrow \frac{1}{T} = \left.\frac{\partial S}{\partial E}\right)_{V,N}, \; \frac{P}{T} = \left.\frac{\partial S}{\partial V}\right)_{E,N}, \; \frac{\mu}{T} = -\left.\frac{\partial S}{\partial N}\right)_{E,N}$

$S = k\ln\Omega \Rightarrow P = kT\left.\frac{\partial \ln\Omega}{\partial V}\right)_{E,N} \Rightarrow \frac{\Omega_2}{\Omega_1} = e^{P\Delta V/kT}$ (same as $e^{\Delta Q/kT}$ where $\Delta Q = P\Delta V$ at const E, N)

$\Omega \uparrow$ as $E\uparrow, V\uparrow, N\uparrow$

$\beta \equiv \frac{1}{V}\left.\frac{\partial V}{\partial T}\right)_P, \quad \kappa \equiv -\frac{1}{V}\left.\frac{\partial V}{\partial P}\right)_T, \quad \frac{1}{\kappa}$ = bulk modulus

coeff. of volume expansion $\quad$ isothermal compressibility

Chapter 11 Summary

The Diffusive Interaction

$$\Delta S_0 = \Delta S_1 + \Delta S_2 = \left(\frac{1}{T_1} - \frac{1}{T_2}\right)\Delta E_1 + \left(\frac{P_1}{T_1} - \frac{P_2}{T_2}\right)\Delta V_1 - \left(\frac{\mu_1}{T_1} - \frac{\mu_2}{T_2}\right)\Delta N_1 \geq 0$$

$\therefore$ at $\equiv^m$, $T_1 = T_2$, $P_1 = P_2$, $\mu_1 = \mu_2$.

approach to $\equiv^m$: $\Delta S_0 > 0 \Rightarrow \left(\frac{1}{T_1} - \frac{1}{T_2}\right)\Delta Q_1 > 0 \Rightarrow$ hot $\to$ cold

$(P_1 - P_2)\Delta V_1 > 0 \Rightarrow$ higher P expands

$-(\mu_1 - \mu_2)\Delta N_1 > 0 \Rightarrow$ flow to lower potential

$$-\frac{\mu}{T} = \left.\frac{\partial S}{\partial N}\right)_{E,V} = k\left.\frac{\partial \ln \Omega}{\partial N}\right)_{E,V} \Rightarrow \frac{\Omega_2}{\Omega_1} = e^{-\mu \Delta N / kT}$$

Chapter 12 Summary

Constraints

$$\Omega_{\text{ideal gas}} \propto E^{\nu/2} = \text{const}\, V^N E^{3N/2} \Rightarrow S_{\text{ideal gas}} = Nk\ln V + \frac{3}{2}Nk\ln E + \text{const.}$$

$$\therefore \left.\frac{\partial S}{\partial E}\right)_{V,N} = \frac{1}{T} \Rightarrow E = \frac{3}{2}NkT, \quad \left.\frac{\partial S}{\partial V}\right)_{E,N} = \frac{P}{T} \Rightarrow \underline{PV = NkT}$$

ideal gas law.

$Nk = nR$

$\therefore PV = nRT$

$Pv = RT$

$v = \frac{V}{n}$ = molar vol.

van der Waals eqn of state

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

c_v & c_p revisited: $dQ = dE + pdV$ (N const)

$$= \frac{\nu}{2}NkdT + pdV \quad (\mu \text{ const})$$

$$\therefore dq = \frac{\nu}{2}RdT + pdv \quad (\text{per mole})$$

$$\therefore c_v = \left.\frac{\partial q}{\partial T}\right)_v = \frac{\nu}{2}R, \quad c_p = \left.\frac{\partial q}{\partial T}\right)_p = \frac{\nu}{2}R + p\left.\frac{\partial v}{\partial T}\right)_p = c_v + p\left.\frac{\partial v}{\partial T}\right)_p$$

ie $c_v = f_n(\nu) \leftarrow$ link between micro & macro.

$p\left.\frac{\partial v}{\partial T}\right)_p = R$ for ideal gas since $pv = RT$

$$\kappa = -\frac{1}{v}\left.\frac{\partial v}{\partial p}\right)_T = -\frac{1}{p}$$

Chapter 13 Summary

Natural Constraints

From perturbation away from Ξ^m, $\Delta S < 0$

$\Rightarrow \Delta\left(\frac{P}{T}\right)\Delta V\Big)_{E,N} < 0 \quad \Rightarrow$ if $V\uparrow$, $P/T \downarrow$

$\Delta T \Delta E\Big|_{V,N} > 0 \quad \Rightarrow$ if $E\uparrow$, $T\uparrow$

$\Delta\left(\frac{\mu}{T}\right)\Delta N\Big|_{E,V} < 0 \quad \Rightarrow$ if $N\uparrow$, $\frac{\mu}{T}\downarrow$

G = Gibbs free energy $\equiv E - TS + pV \; (= \mu N)$ (t/d potential)

F = Helmholtz free energy $\equiv E - TS$

H = Enthalpy $\equiv E + pV$ ← heat function

$dG = -S\,dT + V\,dp + \mu\,dN$

$dF = -S\,dT - p\,dV + \mu\,dN \Rightarrow$

$dH = T\,dS + V\,dp + \mu\,dN$

had $dE = T\,dS - p\,dV + \mu\,dN$

Maxwell's relations

$\frac{\partial E}{\partial S}\Big)_{V,N} = T, \quad \frac{\partial E}{\partial V}\Big)_{S,N} = -p, \quad \frac{\partial E}{\partial N}\Big)_{S,V} = \mu$

$\therefore \frac{\partial}{\partial S}\left(\frac{\partial E}{\partial V}\right) = -\frac{\partial p}{\partial S}\Big)_{V,N} = \frac{\partial^2 E}{\partial S \partial V}$

$+ \frac{\partial}{\partial V}\left(\frac{\partial E}{\partial S}\right) = \frac{\partial T}{\partial V}\Big)_{S,N} = \frac{\partial^2 E}{\partial V \partial S}$ =

$\therefore -\frac{\partial p}{\partial S}\Big)_{V,N} = \frac{\partial T}{\partial V}\Big)_{S,N}$, etc

48 relations in all.

Measurements: T & p, $C_{V,p} = \frac{\partial Q}{\partial T}\Big)_{V,p} = T\frac{\partial S}{\partial T}\Big)_{V,p}$, $\beta = \frac{1}{V}\frac{\partial V}{\partial T}\Big)_p$, $\kappa = -\frac{1}{V}\frac{\partial V}{\partial p}\Big)_T$

μ is elusive → $\Delta\mu = \frac{\Delta E}{N} - \frac{\nu}{2}k\Delta T$ or use electron affinities

Applications of Maxwell's relations:

$\Delta E = (\underbrace{C_p - PV\beta}_{\frac{\nu}{2}k\Delta T})\Delta T - (\underbrace{T\beta - P\kappa}_{= 0 \text{ if } \Delta E = \frac{\nu}{2}k\Delta T})V\Delta P \quad (\Delta N = 0)$

True if ideal gas: → for example.

$(PV = NRT)$ since $\kappa = -\frac{1}{V}\frac{\partial V}{\partial P}\Big)_T = \frac{NKT}{P^2V}$ & $\beta = \frac{1}{V}\frac{\partial V}{\partial T}\Big)_P = \frac{NR}{PV}$

degree of violation $= \frac{(T\beta - P\kappa)V\Delta P}{(C_p - PV\beta)T}$

Ideal Gas: $C_V = 3/2\,\underbrace{NK}_{nR}$, $C_p = 5/2\,NK$

$C \to 0$ as $T \to 0$

Chapter 14 Summary

Imposed Constraints

Possibilities: Std N, dV, dQ, dP or dT to 0.

Approach to choice of variables:

1. What property wish to study?
2. What are the constraints?
3. What can we measure?

Example: Heat water: $dE = \frac{\partial E}{\partial S}\Big)_{N,p} \frac{dQ}{T} \Rightarrow dE = \left(T - p\frac{\partial V}{\partial S}\right)_{p,N} \frac{dQ}{T}$

($dP=0$) Isobaric: $dH = TdS$ if $dN=0$ as well ← heat function

$\beta = \frac{1}{V}\frac{\partial V}{\partial T}\Big)_p$

($dT=0$) Isothermal: $dF = -pdV$ if $dN=0$ as well ← work function

$\kappa = -\frac{1}{V}\frac{\partial V}{\partial p}\Big)_T$

($dQ=0$) Adiabatic: $dE = -pdV$, $E = \frac{\nu}{2}NkT$, $PV = NkT$ (Ideal gas) $\Rightarrow PV^{\gamma} = \text{const}$

$\hookrightarrow dS = 0$ at $\equiv^m$

($+\ TV^{\gamma-1} = \text{const}$) $\quad \gamma = \frac{\nu+2}{\nu}$

$$\frac{C_p}{C_V} = \frac{(\nu/2+1)Nk}{\nu/2\,Nk} = \frac{\nu+2}{\nu} = \gamma$$

Solids & liq: $dS = \frac{\partial S}{\partial T}\Big)_V dT + \frac{\partial S}{\partial V}\Big)_T dV = 0 \Rightarrow \frac{dT}{T} = -\frac{\beta}{\kappa C_V} dV$

$= \frac{\partial S}{\partial T}\Big)_p dT + \frac{\partial S}{\partial p}\Big)_T dp = 0 \Rightarrow \frac{dT}{T} = \frac{V\beta}{C_p} dp \Rightarrow \frac{dV}{V} = -\frac{\kappa C_V}{C_p} dp$

Reversible process: must have $\Delta S = 0$ to get back.

Joule Thompson (Throttling) Process: $H = E + PV = \text{const}$, $\Delta H = 0$

$\Delta Q = 0 = \Delta E + \Delta W$

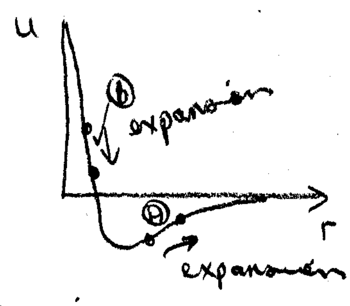
$\Delta S \neq Q\Delta T$ since not $\equiv^m$

(Used for gas cooling).

$dT = \frac{-V}{C_p}(1-\beta T)dP$

Free Expansion

$\Delta E = \Delta \underset{0}{Q} - \Delta \underset{0}{W} = 0$, $dE = 0 = TdS - pdV \Rightarrow dS = \frac{p}{T}dV$



$dT = \frac{1}{C_V}\left[p - T\frac{\partial P}{\partial T}\Big)_V\right] dV$
$= 0$ for ideal gas.

$E = \overbrace{\mu + \tfrac{1}{2}kr^2}^{u} + \tfrac{1}{2}mv^2$

H$_2$ explosions

Refrigeration

A $u \uparrow$ when $V\uparrow$ ∴ KE $\downarrow$ ∴ T $\downarrow$

B $u \downarrow$ " $V\uparrow$ ∴ KE $\uparrow$ ∴ T $\uparrow$

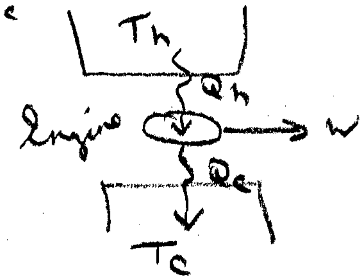
Chapter 15 Summary

Engines + Refrigerators

Isothermal: $PV = nRT$, $T = \text{const}$ $\therefore PV = \text{const}$ + $\Delta E = \frac{1}{2} k \nu N \Delta T = 0$

$$\Delta W = \int_{V_i}^{V_f} P\,dV = nRT \ln(V_f/V_i) = \Delta Q \quad \text{since } \Delta E = 0$$

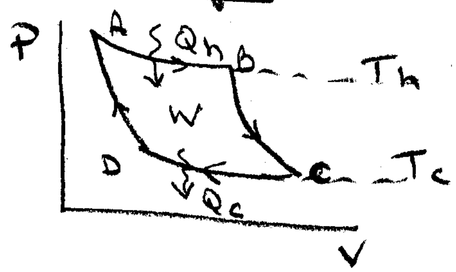
Heat Engine $\Delta E = 0 \Rightarrow W = Q_h - Q_c$



$$e \equiv \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

Heat Pump + refrigerator has arrows reversed

Carnot Cycle



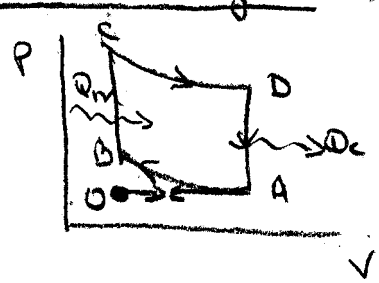
$A \to B$ Isothermal expansion
$Q_h = W_{AB} = nRT_h \ln(V_B/V_A)$
$B \to C$ Adiabatic expansion $T_h V_B^{\gamma-1} = T_c V_C^{\gamma-1}$
$C \to D$ ~~Isothermal~~ Isothermal compression
$Q_c = -W_{CD} = nRT_c \ln(V_C/V_D)$
$D \to A$ Adiabatic compression
$T_h V_A^{\gamma-1} = T_c V_D^{\gamma-1}$

Can use Carnot cycle to measure temperature.

$$\therefore \frac{V_B}{V_A} = \frac{V_C}{V_D} \Rightarrow \frac{Q_c}{Q_h} = \frac{T_c}{T_h}$$

$\therefore e = 1 - T_c/T_h$ for Carnot

Gasoline Engine: Otto cycle



$O \to A$ intake
$A \to B$ Adiabatic Compression, $T\uparrow$
$B \to C$ Combustion, Q_h added, $T\uparrow\uparrow$
$C \to D$ Power stroke, adiabatic expansion
$D \to A$ Exhaust valves open, Q_c removed
$A \to O$ Exhaust stroke

$$e = 1 - \frac{Q_c \Leftarrow nC_v(T_C - T_B)}{Q_h \Leftarrow nC_v(T_D - T_A)}$$

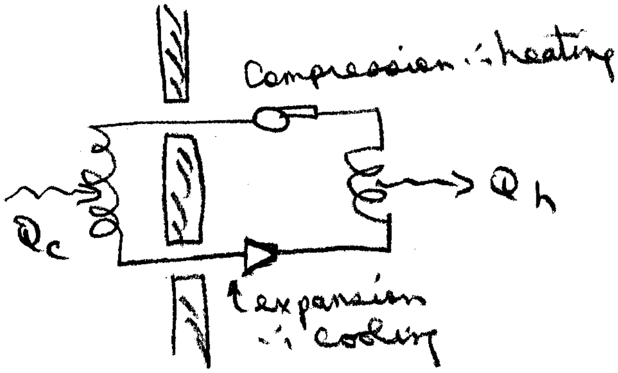
$$= 1 - \frac{(T_D - T_A)}{(T_C - T_B)} \Rightarrow 1 - \frac{1}{(V_1/V_2)^{\gamma-1}}, \quad e\uparrow \text{ as } CR\uparrow$$

Compression Ratio

$$= 1 - \frac{T_D}{T_C}$$ lower e of Carnot

Chapter 15 Summary (cont'd)

Heat Pumps + Refrigerators



Coeff of Performance ≡ COP
(heat Pump)

$$= \frac{Q_h}{W} = \frac{Q_h}{Q_h - Q_c}$$

$$= \frac{T_h}{T_h - T_c} \quad \text{carnot}$$

COP (refrigerator)

$$= \frac{Q_c}{W} = \frac{T_c}{T_h - T_c}$$