ENGINEERING 2C3 Assignment #5 Solution

1. [Serway Chapter 20 Problem 2, pg 577]
An 80 kg weight watcher wishes to climb a mountain to work off the equivalent of a large piece of chocolate cake rated at 700 (food) Calories. How high must the person climb?

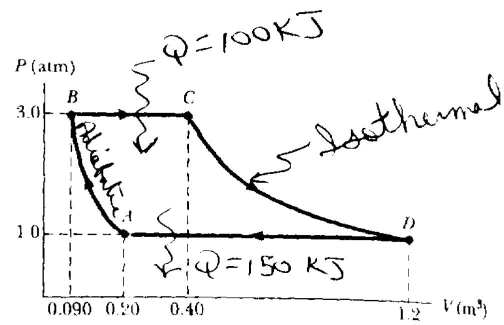
$$W = mgh = 700 \times 10^3 \text{ calories}$$
$$= 700 \times 10^3 \times 4.186 \text{ J}$$
$$\therefore h = \frac{700 \times 10^3 \times 4.186 \text{ J}}{80 \text{ Kg} \times 9.8 \text{ m/s}^2} = 3737.15 \text{ m}.$$

2. [Serway Chapter 20 Problem 15, pg 578]
A water heater is operated by solar power. If the solar collector has an area of 6.0 m^2 and the power delivered by sunlight is 550 W/m^2, how long does it take to increase the temperature of 1.0 m^3 of water from 20 °C to 60 °C?

$$Q = 550 \text{ W/m}^2 \times 6 \text{ m}^2 \times t = mc\Delta T = 1 \text{ m}^3 \times 1000 \frac{\text{kg}}{\text{m}^3} \times 4186 \frac{\text{J}}{\text{Kg}\,^\circ\text{C}} \times 40\,^\circ\text{C}$$
$$\therefore t = \frac{1000 \times 4186 \times 40}{550 \times 6} = 50{,}740 \text{ s} = 14.09 \text{ hr.}$$

3. [Serway Chapter 20 Problem 40, pg 580]
An ideal gas system goes through the process shown in the figure.
- From A to B, the process is adiabatic.
- From B to C, the process is isobaric with 100 kJ of heat flowing into the system.
- From C to D, the process is isothermal.
- From D to A, the process is isobaric with 150 kJ of heat flowing out of the system.

Determine the difference in internal energy $U_B - U_A$.

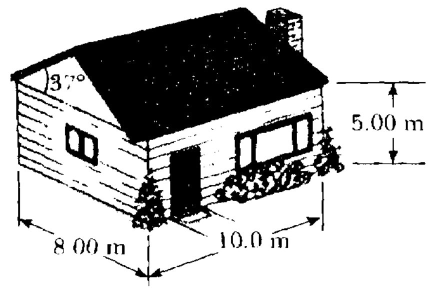


$$U_A - U_B = (Q - W)_{B\to C} + (Q - W)_{C\to D} + (Q - W)_{D\to A}$$
$$= 100 - (P\Delta V)_{BC} + 0 + (-150 - (P\Delta V)_{D\to A})$$

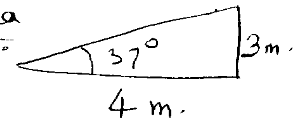
heat added just compensates for expansion work, ie $PV = nRT$ = constant

$$= 100 - (3 \times 101 \times 10^3 \text{ Pa})(0.31 \text{ m}^3) - 150 \times 10^3 - (101 \times 10^3 \text{ Pa})(-1.0 \text{ m}^3)$$
$$= -42.93 \text{ kJ} \qquad \therefore U_B - U_A = 42.93 \text{ kJ}$$

4. [Serway Chapter 20 Problem 72, pg 583]
The average thermal conductivity of the walls (including the windows) and roof of the house shown is 0.48 W / m °C, and their average thickness is 21 cm. The house is heated with natural gas having a heat of combustion (heat given off per cubic meter of gas burned) of 9600 kcal / m^3. How many cubic meters of gas must be burned each day to maintain an inside temperature of 25.0 °C if the outside temperature is 0.0 °C? Disregard radiation and heat loss through the ground.



Roof area



$\cos 37° = \frac{4}{L} = 0.798$
$\therefore L = 5.0\,m.$

(ive height must be 3 m since it is a 3-4-5 triangle)

Heat transfer area = $\underbrace{2\times5\times8}_{\text{side walls}} + \underbrace{2\times5\times10}_{\text{main walls}} + \underbrace{2\times5\times10}_{\text{roof}} + \underbrace{2\times3\times4}_{\text{eves}}$

$= 304.\,m^2$

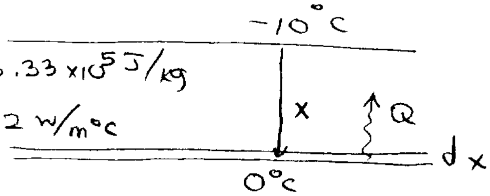
$$H = -KA\frac{\Delta T}{\Delta x} = Q/\Delta t$$

$$\therefore Q = -0.48\,J/s\,m°c \times 304\,m^2 \times \frac{(-25°c)}{0.21\,m} \times 1\,day \times \frac{60\times60\times24\,s}{day}$$

$$= 1.500\times10^9\,J/day \Rightarrow \therefore m^3\,gas = \frac{1.500\times10^9\,J}{9.6\times4.186\times10^6\,J/m^3} = 37.33\,m^3$$

5. [Serway Chapter 20 Problem 82, pg 584]
A pond of water at 0 °C is covered with a layer of ice 4.0 cm thick. If the air temperature stays constant at -10 °C, how long will it take before the ice thickens to 8.0 cm? (Hint: To solve this problem, utilize dQ/dt = kA ΔT/x and note that the incremental heat dQ extracted from the water through the thickness x of ice is the amount required to freeze a thickness dx of ice. That is dQ = LρAdx, where ρ is the density of the ice, A is the area, and L is the latent heat of freezing.)

$L = 3.33\times10^5\,J/kg$
$k = 2\,W/m°c$



To freeze thickness dx:
$dQ = \underbrace{\rho A dx}_{\text{mass}} \times L$

This energy is conducted from this mass to the air through the existing layer of ice, ie $\frac{dQ}{dt} = -kA\frac{\Delta T}{x}$

$\therefore \rho A dx L = \frac{kA(-\Delta T)}{x}dt \Rightarrow dt = \left(\frac{\rho L}{k(-\Delta T)}\right)x\,dx$ (seconds)

$$\therefore t = \frac{\rho L}{k(-\Delta T)}\int_{.04}^{.08} x\,dx = \frac{\rho L}{k(-\Delta T)}\left(\frac{64}{2} - \frac{16}{2}\right)\times10^{-4}$$

$$= \frac{917\,\frac{kg}{m^3}\times3.33\times10^5\,J/kg\times24\times10^{-4}\,m^2}{2\,W/m°c\times10°c} = 3.66\times10^4\,sec = 10.18\,hr.$$