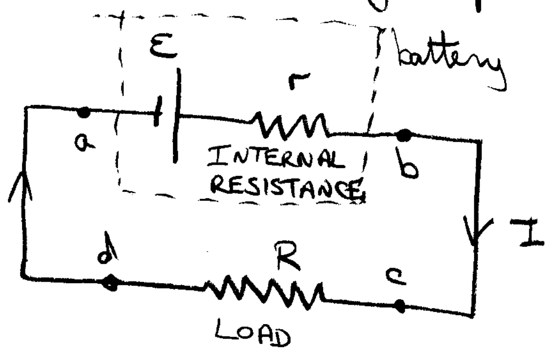
Chapter 28 Direct Current Circuits

28.1 Electromotive Force (emf)

Source of energy: emf. Produces electric field
eg: battery, generator
↓
charge motion ie current.

Like a charge "pump".



$$V = V_b - V_a = \varepsilon - Ir$$
$$= IR$$
$$\therefore \quad \varepsilon = IR + Ir = I(R+r)$$
$$\therefore \quad I = \frac{\varepsilon}{R+r}$$

Power delivered by battery $= I\varepsilon = I^2(R+r)$

Example Terminal Voltage of a Battery

Battery: $\varepsilon = 12.0\,V$, $r = 0.05\,\Omega$

Load: $3.00\,\Omega$.

What is I and terminal voltage of the battery?
Power dissipated in load? In r? Total?

$$I = \frac{\varepsilon}{R+r} = \frac{12.0\ V}{3.05\ \Omega} = 3.93\ A$$

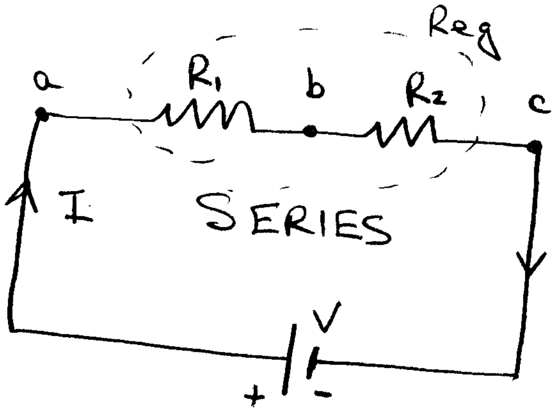
$$V = \varepsilon - Ir = 12.0 - 3.93 \times 0.05 = 11.8\ V$$

$$P_R = I^2R = (3.93)^2(3.00) = 46.3\ W$$

$$P_r = I^2 r = (3.93)^2(0.05) = 0.772\ W$$

$$P_{Total} = P_r + P_R = 47.1\ W = \underline{I\varepsilon}$$

28.2 Resistors in Series and in Parallel



$$V = IR_1 + IR_2$$
$$= I(R_1 + R_2)$$
$$= I \underbrace{R_{eq}}_{\text{equivalent resistance}}$$

In general:

$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

Power in each R: $\quad I^2 R_1 \;\text{ vs }\; I^2 R_2$

If R_1 were a "60 W" bulb, ie it dissipates 60 W when V=120,

$$P_{R_1} = IV = \frac{V^2}{R_1} = \frac{(120)^2}{R_1} = 60 \quad \text{if wired separately}$$

$$\therefore R_1 = \frac{(120)^2}{60} = 240\,\Omega.$$

For a "120 W" bulb, $R_1 = \frac{(120)^2}{120} = 120\,\Omega.$

$\therefore$ If a "60W" bulb and a "120 W" bulb were wired in series to a 120 V source,

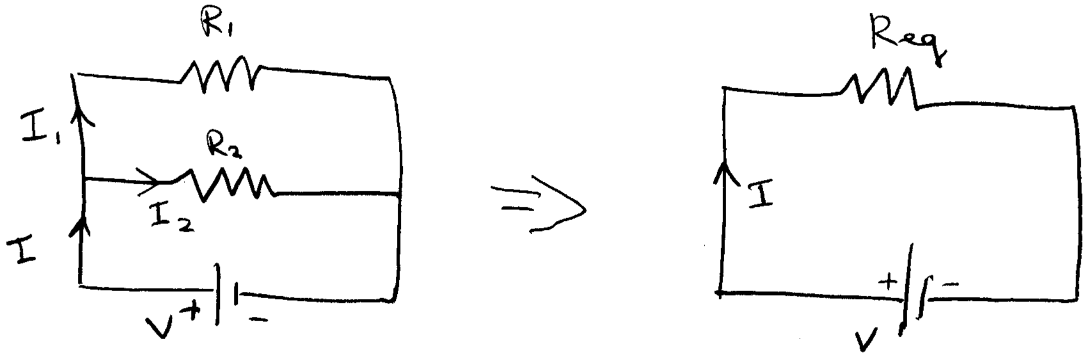
$$I = \frac{120}{(240+120)} = \frac{120}{360} = \tfrac{1}{3}\,A.$$

$$\therefore P_{R_1} = I^2 R_1 = \left(\tfrac{1}{3}\right)^2 \times 240 = \frac{240}{9} = 26.\dot{6}\ W \Leftrightarrow 60W$$

$$P_{R_2} = I^2 R_2 = \left(\tfrac{1}{3}\right)^2 \times 120 = \frac{120}{9} = \underline{13.\dot{3}\ W} \Leftrightarrow 120\,W$$

$P_{Total} = 40$ W

$R_{eq} = 240 + 120 = 360\,\Omega. \Rightarrow P_{Total} = I^2 R_{eq} = \left(\tfrac{1}{3}\right)^2 \times 360 = 40\,W$



$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_{eq}}$$

$$\therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

or in general: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

If R_1 were a "60 W" bulb, the power dissipated when $V = 120\,V$ is $\frac{V^2}{R_1} = \frac{(120)^2}{240} = 60W$ (duh!)

$\uparrow$ from before

If R_2 were a "120W" bulb, the power dissipated is 120W. $\Rightarrow$ $P_{Total} = 60 + 120 = 180\,W$

$$I_1 = \frac{V}{R_1} = \frac{120}{240} = 0.5\,A$$

$$I_2 = \frac{V}{R_2} = \frac{120}{120} = 1.0\,A$$

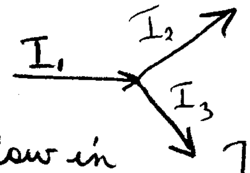
$$I = I_1 + I_2 = 1.5\,A$$

$$\frac{1}{R_{eq}} = \frac{1}{240} + \frac{1}{120} = \frac{3}{240} \Rightarrow R_{eq} = 80\,\Omega.$$

$$\left.\begin{aligned} P_{Total} &= I\,V = 1.5 \times 120 = 180\,W \\ &= \frac{V^2}{R_{eq}} = (120)^2/80 = 180\,W \end{aligned}\right\}$$

28.3 Kirchhoff's Rules

1. $\sum_j I_j = 0$ at a junction



CONSERVATION OF CHARGE

ie $I_1 - I_2 - I_3 = 0$ [note: +ve if flow in, −ve if flow out.]

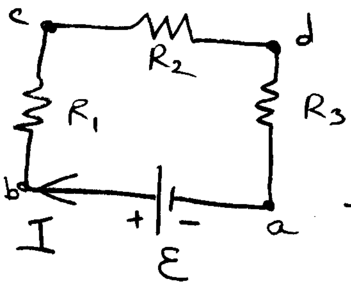
or $I_1 = I_2 + I_3$ (flow in = flow out)

Pick a current direction and stick with it for the duration of the problem.

2. $\sum_j \Delta V_j = 0$ around a loop

↑ changes in potential energy

CONSERVATION OF ENERGY



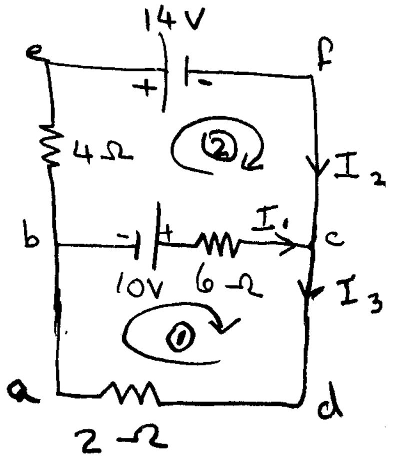
ie $+\mathcal{E} - IR_1 - IR_2 - IR_3 = 0$

or $\mathcal{E} = IR_1 + IR_2 + IR_3$

Voltage change $= -IR_3$ ie it drops

Note: in steady state, a capacitor is an open circuit since charge builds up to just balance the potential difference across it.

Example: Applying Kirchhoff's Rules



Junction c

$I_1 + I_2 = I_3$

(junction b doesn't add anything new)

Loop ①

$10 - 6I_1 - 2I_3 = 0$

Loop ②

$-14 + 6I_1 - 10 - 4I_2 = 0$

Have 3 eqn's with 3 unknowns

Eqn 1 $\quad I_1 + I_2 - I_3 = 0$

Eqn 2 $\quad -6I_1 \qquad - 2I_3 = -10$

Eqn 3 $\quad 6I_1 - 4I_2 \qquad = 24$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 6 & 0 & 2 \\ 6 & -4 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 24 \end{bmatrix}$$

[<u>Matlab</u> gives $I_1 = 2A$, $I_2 = -3A$, $I_3 = -1A$.]

or manually:

Eqn 1 → Eqn 2 : $+6I_1 + 2(I_1 + I_2) = +10$

$\therefore 8I_1 + 2I_2 = 10$

(Eqn 3)/2 + Eqn 2 :

$$\begin{aligned} 3I_1 - 2I_2 &= 12 \\ 8I_1 + 2I_2 &= 10 \\ \hline 11 I_1 &= 21 \Rightarrow \underline{\underline{I_1 = 2A}} \end{aligned}$$

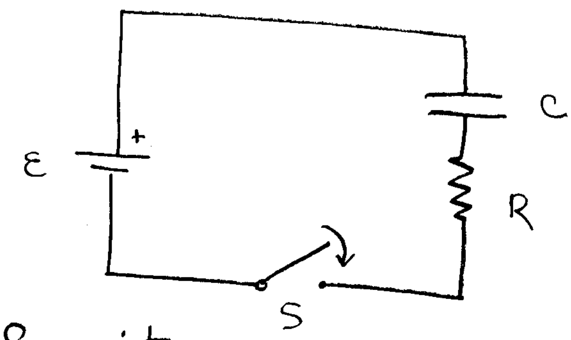
$\therefore$ from Eqn 3 : $I_2 = \dfrac{12-24}{4} = \underline{\underline{-3A}}$

+ from Eqn 1 : $I_3 = 2 - 3 = \underline{\underline{-1A}}$

28.4 RC Circuits

Charging:

After the switch is closed:



$$\sum_j \Delta V_j = 0$$

ie $\mathcal{E} - \dfrac{q}{C} - IR = 0$ [C = Capacitance – see note below]

note: $q = q(t)$, $I = I(t)$.

at $t=0$, no charge has accumulated on C,

$\therefore\ I(0) = \dfrac{\mathcal{E}}{R}$

at $t=\infty$, the charge has built up on C causing $I(\infty) = 0$.

This implies that $\mathcal{E} = q(\infty)/C$ at $t = \infty$.

Since $I \equiv \dfrac{dq}{dt} \Rightarrow \mathcal{E} - \dfrac{q}{C} - \dfrac{dq}{dt}R = 0$

$$\Rightarrow \frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} \quad \Longleftarrow \text{Need to solve.}$$

Note: Recall (Serway 26.2) for a parallel plate capacitor:

$E = \dfrac{\sigma}{\epsilon_0} = \dfrac{q}{\epsilon_0 A}$ (from Gauss' Law $\oint E \cdot dA = q/\epsilon_0$)

$+ V = Ed = \dfrac{qd}{\epsilon_0 A} \equiv q/C$

To solve, Let $q = y + \mathcal{E}C$

Thus: $\dfrac{dy}{dt} = \dfrac{\mathcal{E}}{R} - y/_{RC} - \dfrac{\mathcal{E}\cancel{C}}{R\cancel{C}}$

$= -y/_{RC}$

$\therefore \dfrac{dy}{y} = -\dfrac{dt}{RC} \equiv -dt/\tau \qquad (\tau \equiv RC)$

$\left[\equiv \dfrac{\text{Volts}}{\text{current}} \times \dfrac{Q}{V}\right]$

$\left[= \dfrac{Q}{Q/T} = \text{time}\right]$

$\therefore \displaystyle\int_{y_0}^{y} \frac{dy}{dt} = -\int_0^t dt/\tau$

$y_0 = q_0 - \mathcal{E}C = -\mathcal{E}C$ (no initial charge)

$\therefore \ln(y/y_0) = -t/\tau \Rightarrow y = y_0 e^{-t/\tau}$

$= -\mathcal{E}C e^{-t/\tau}$

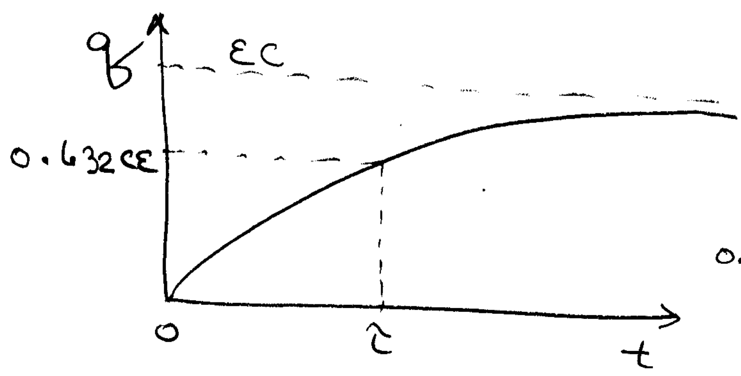
$\therefore q = \mathcal{E}C(1 - e^{-t/\tau})$

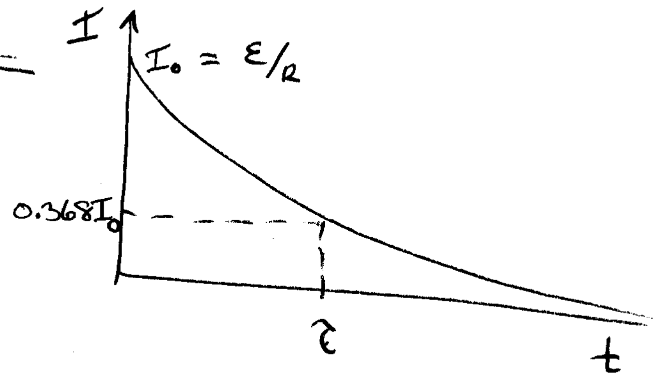
$= q(0)(1 - e^{-t/\tau})$

Since $I = dq/_{dt} \Rightarrow I = \mathcal{E}C(-e^{-t/\tau})(-1/\tau)$

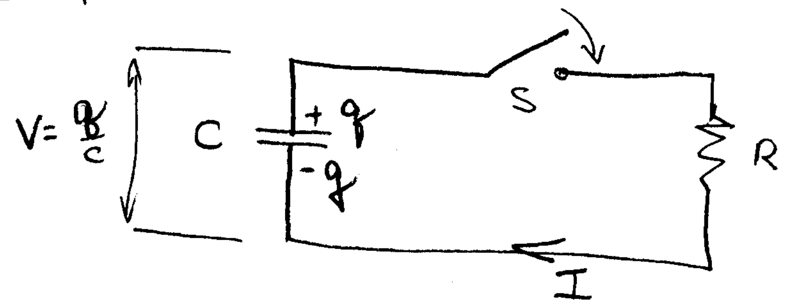
$\therefore I = \mathcal{E}/_R \, e^{-t/\tau}$

$= \mathcal{E}/_R \, e^{-t/RC}$





Discharging a charged Capacitor



Since $\sum_j{}' \Delta V_j = 0$,

$$IR = q/c \quad , \quad I = I(t), \quad q = q(t)$$

now $I = -\dfrac{dq}{dt}$ (since I = rate of decrease of charge on capacitor. I is still $\dfrac{dq_{wire}}{dt}$ where q_{wire} = charge flowing through the wire.)

$$\therefore -R\frac{dq}{dt} = q/c \Rightarrow dq/q = -\frac{1}{RC}dt$$

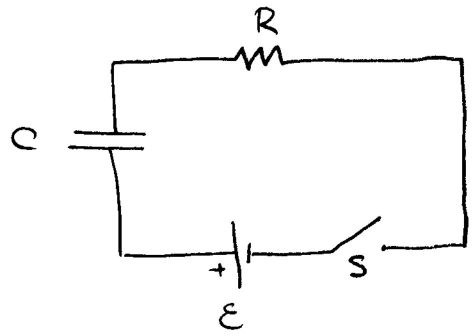
Thus: $$\int_{q(0)}^{q} \frac{dq}{q} = -\int_0^t \frac{dt}{RC}$$

$$\Rightarrow q = q(0)\,e^{-t/RC} = q(0)\,e^{-t/\tau}$$

$$\& \ I = -\frac{dq}{dt} = \frac{q(0)}{RC}e^{-t/\tau} = I(0)\,e^{-t/\tau}$$

decay just like the charging case.

Example: Charging a Capacitor in an RC Circuit



$\mathcal{E} = 12\ V$
$C = 5\ \mu F$
$R = 8 \times 10^5\ \Omega$

Find τ, q_{max}, I_{max}, $q(t)$, $I(t)$

$\tau = RC = 8 \times 10^5\ \Omega \times 5 \times 10^{-6}\ F = 4.0\ s$

$q_{max} = C\mathcal{E} = 5 \times 10^{-6} F \times 12 V = 60 \mu C$ (coulombs)

$I_{max} = \frac{\mathcal{E}}{R} = \frac{12 V}{8 \times 10^5 \Omega} = 15 \mu A$

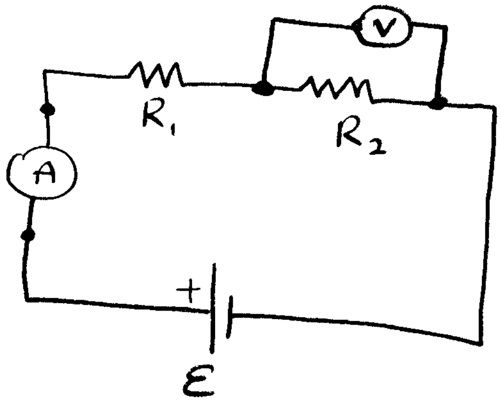
$\therefore\ q(t) = 60 [1 - e^{-t/4}]\ \mu C$

$I(t) = 15\ e^{-t/4}\ \mu A$

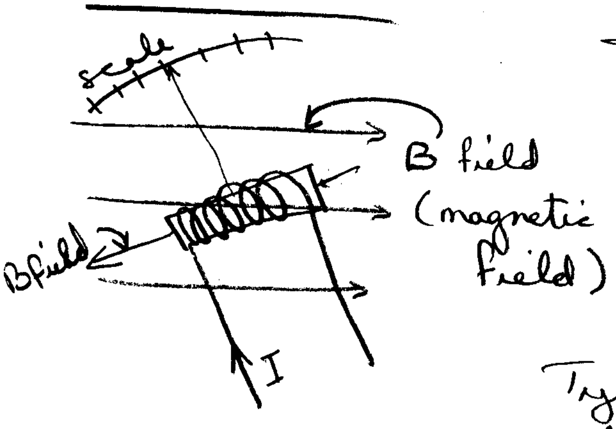
28.5 Electrical Instruments

Ammeter
- measures current.
- wire in series
- ideally meter resistance = 0 so as to not disturb the current.
- in reality, $\Omega_{meter} \ll R_{circuit}$

Voltmeter
- measures voltage
- wired in parallel to component
- ideally meter resistance = ∞ so no current passes through it.
- in reality $\Omega_{meter} \gg R_{component}$



Galvanometer

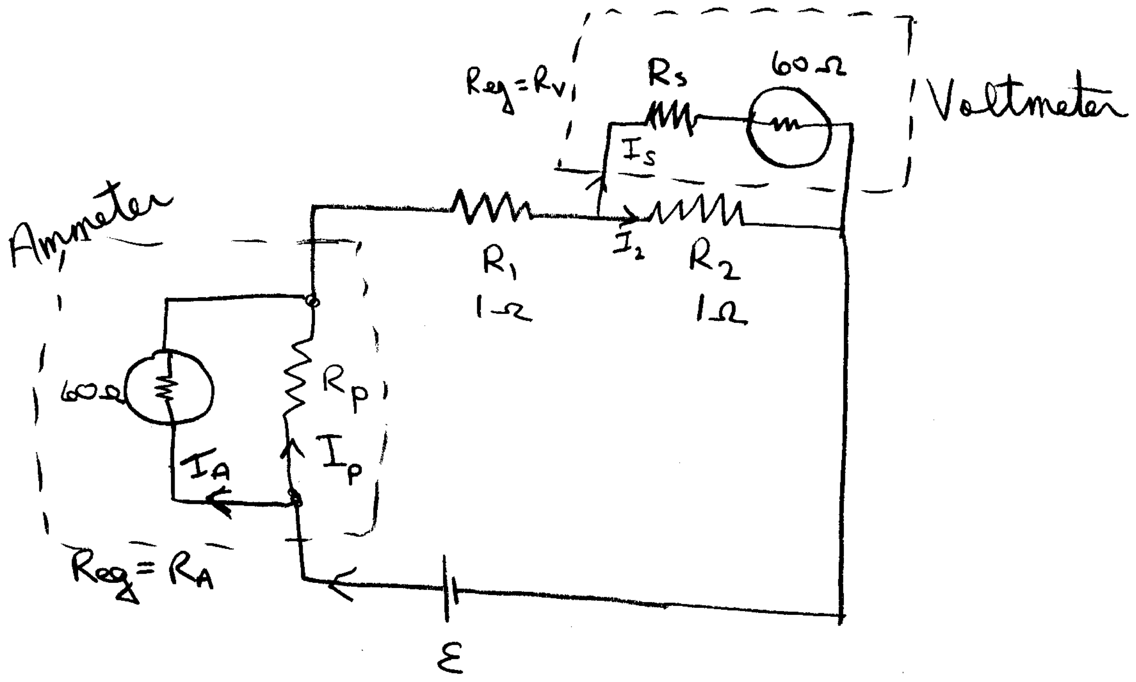


The coil is mounted on a torsional spring. Current loop makes a magnetic field $\propto$ to current.

Force $\propto B_{field} \propto I$

$\therefore$ measurement $\propto I$

Typical Ω of coil $\sim 60\,\Omega$.
Get full scale movement for $I \sim 1\,mA$.



Let's say: $R_1 = R_2 = 1\,\Omega \quad \therefore \quad I = \frac{\mathcal{E}}{2}\,A$

Ammeter
~~If~~ just have the ammeter hooked up:
Want R_p small compared to $2\,\Omega$

Say Ammeter shows full scale at 1 mA. Pick $R_p = 0.01\,\Omega$.

Thus $R_{eq} = R_A = \dfrac{1}{\frac{1}{60} + \frac{1}{.01}} = 0.00999 \simeq .01\,\Omega$

$\therefore \quad I_{Actual} = \dfrac{\mathcal{E}}{2.01}\,A = I_A + I_p$

1 part in 200 error $= 0.5\%$

$+ \quad \Delta V_{ammeter} = I_A R_A = I_p R_p$

$\therefore \quad \dfrac{I_A}{I_p} = \dfrac{R_p}{R_A} = \dfrac{.01}{60} = .000167$ ie $I_A \ll I_p$

$\therefore \quad \dfrac{\mathcal{E}}{2.01} = I_A + \dfrac{60}{.01} I_A$

$= 6001\, I_A \implies I_A = 8.29 \times 10^{-5}\, \mathcal{E}\, A$

$\therefore$ scale $= I_{measured} \times \dfrac{I_{actual}}{I_A} = I_{meas} \times \dfrac{\mathcal{E}/2.01}{8.29 \times 10^{-5}\,\mathcal{E}} = 6001\, I_{meas.}$

Voltmeter:

If just have the voltmeter hooked up:

Want R_s large compared to R_2, say $940\,\Omega$.

Then $R_{eq} = R_V = R_s + 60 = 1000\,\Omega$.

Thus the total resistance of the circuit is:

$$R_{Total} = R_1 + \frac{1}{\frac{1}{1000} + \frac{1}{1}} = 1 + \frac{1}{1.001} = 1.999\,\Omega.$$

$$\therefore I_{Actual} = \frac{\mathcal{E}}{1.999\,\Omega}$$

Now $\Delta V_{voltmeter} = I_s(R_s+60) = I_2 R_2$

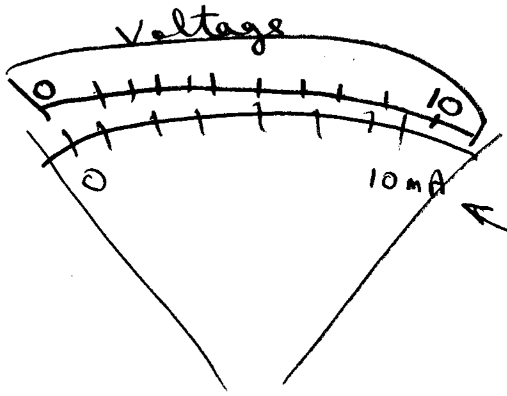
$$\therefore \frac{I_s}{I_2} = \frac{R_2}{R_s+60} = \frac{1}{10.00}$$

and $I_{Actual} = I_s + I_2 = I_s + 10.00 I_s = 1001\, I_s$

$$\therefore I_s = \frac{\mathcal{E}}{1.999 \times 1001} \qquad = 1.001\, I_2$$

Voltage across $R_2 = I_s(R_s + 60) = \dfrac{1000\,\mathcal{E}}{1.999 \times 1001} = 0.49975\,\mathcal{E}$

$\therefore$ Scale $= I_{measured} \times \dfrac{V_{R_2}}{I_s} = \dfrac{I_s(R_s+60)}{I_s} = I_{meas.} \times 1000$



Say the galvanometer reads full scale when it passes a current of 10 mA

Error

$\sim 0.1\%$

(ie $I_{meas.} \sim 1.001\, I_2$)

Of course, in general you won't know what R_1 & R_2 are, so you can only guess at the real error.

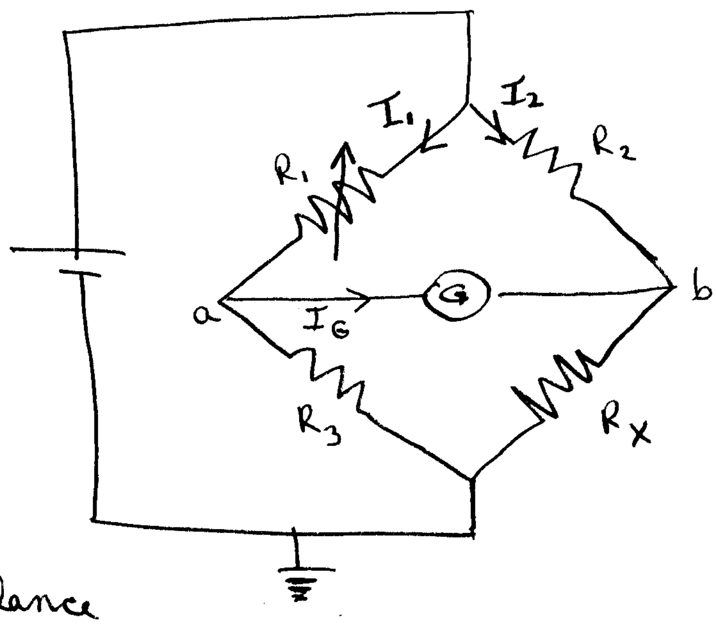
But as the previous example shows,

for an ammeter, a shunt resistance of 1/200 of the circuit resistance gives an error of ~0.5%. If the circuit resistance were higher, the error would drop. Conversely, a low resistance circuit would cause large errors with this 60 Ω galvanometer movement.

For the voltmeter, a shunt resistance of ~1000 times the load resistance gives an error ~0.1%. For highly resistive loads, the errors would rise.

Wheatstone Bridge

Used to measure an unknown resistor, R_x.



Vary R_1 until $I_G = 0$

Thus $I_1 R_1 = I_2 R_2$

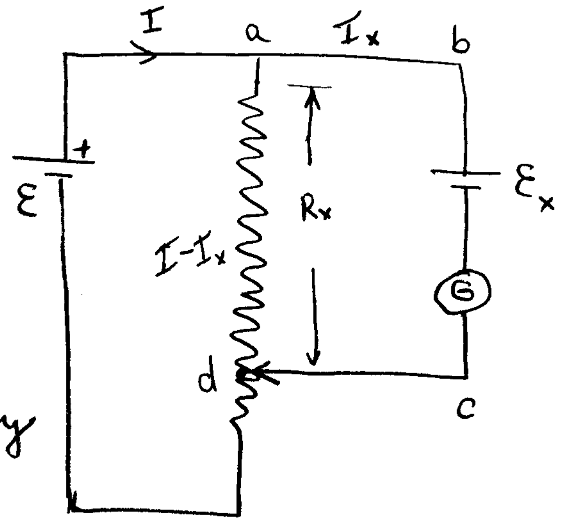
\+ $I_1 R_3 = I_2 R_x$

Since $V_a = V_b$ at the balance condition.

$$\therefore \quad R_x = \frac{R_2 R_3}{R_1}$$

Potentiometer

Used to measure an unknown $\mathcal{E}_x$.



Slide R_x until $I_x = 0$.

Thus $\mathcal{E}_x = I R_x$.

But you don't know I accurately since $\mathcal{E}$ has some internal R

So replace $\mathcal{E}_x$ with a standard.

Repeat to give $\mathcal{E}_s = I R_s$

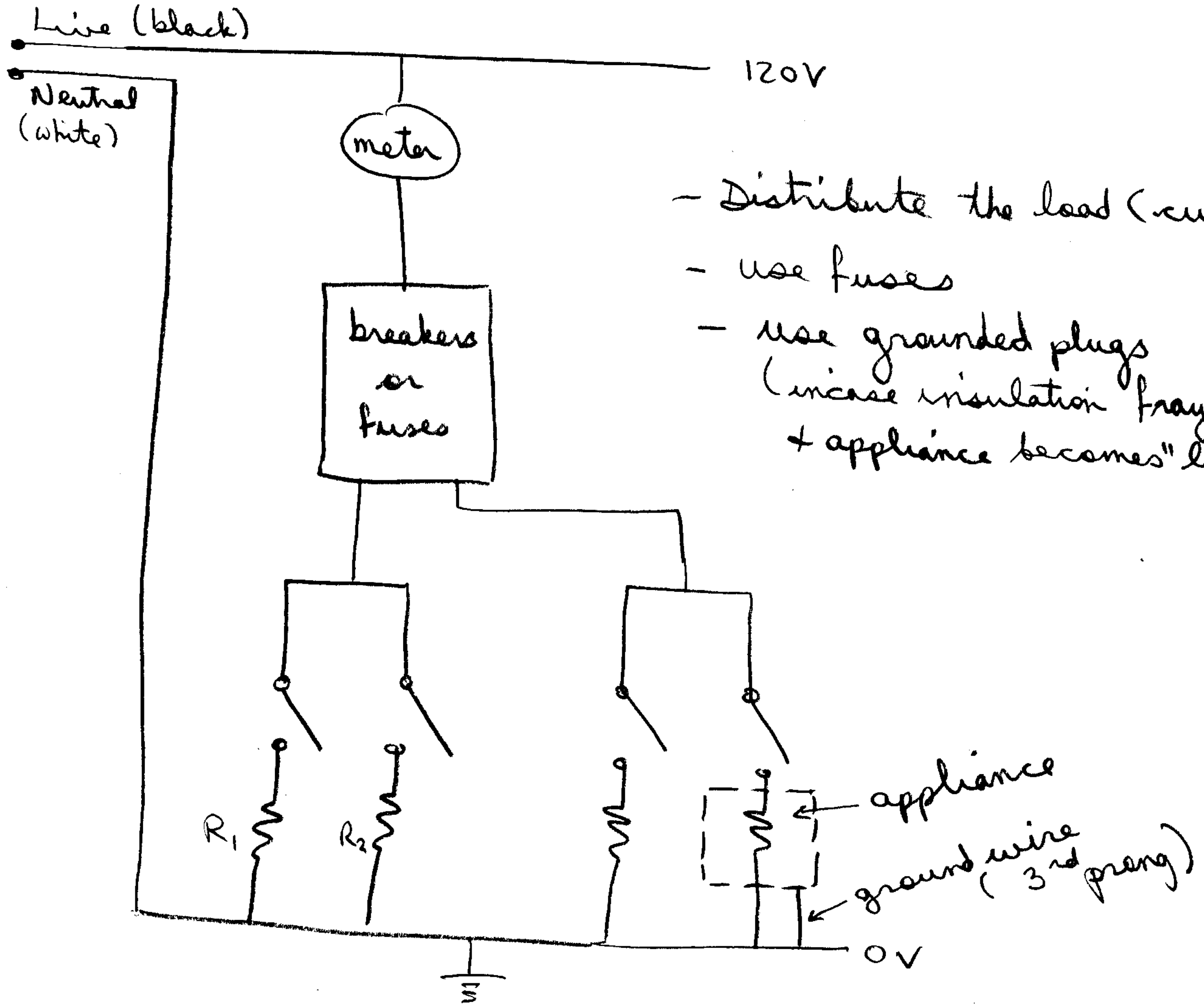
$$\therefore \quad \mathcal{E}_x = \frac{R_x}{R_s} \cdot \mathcal{E}_s$$

known Emf at zero current.

Since $R \propto$ length, L, on a sliding wire,

we have $\mathcal{E}_x = \dfrac{L_x}{L_s} \cdot \mathcal{E}_s$.

28.6 Household Wiring and Electrical Safety



- Distribute the load (current)
- use fuses
- use grounded plugs (incase insulation frays + appliance becomes "live")