Chapter 27 Current + Resistance

27.1 Current

Symbol: I or i = current

Unit : ampere (A)

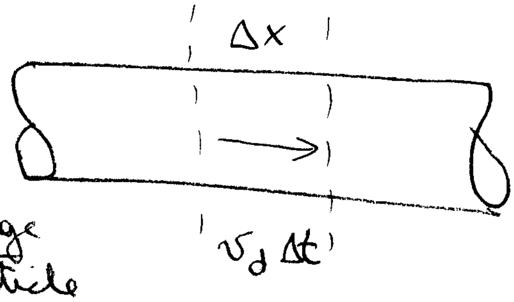
Definition : $I \equiv \frac{dQ}{dt}$ ← Q = charge, units of coulomb (C)

$$1\,A = \frac{1\,C}{1\,s}$$

A single electron has a charge of 1.60219×10^{-19} C

Convention: current is +ve when flow of charge is +ve,
∴ current flows in opposite direction to electrons.

Flow of charged carriers



$\Delta Q = \#$ of particles × $\frac{\text{charge}}{\text{particle}}$

$= (n \cdot \text{Area} \cdot \Delta x)\, q$, $\Delta x = v_d \Delta t$

n ← number density ; v_d ← drift velocity

$= n\, \text{Area}\, v_d\, \Delta t\, q$

$\therefore\ I = n q v_d\, \text{Area}$

27.2 Resistance and Ohm's Law

Symbol: $\bar{J}$ = current density (vector quantity)
unit: A/m^2
~~De~~finition: $\bar{J} \equiv I/_{Area} = nq\bar{v}_d$ (iff J uniform over Area)

Recall from 1st year physics:

$\bar{J} = \sigma \bar{E}$ = conductivity × electric field

This is a statement of <u>Ohm's law</u>.

Note: linear relationship between $\bar{J}$ + $\bar{E}$

<u>In more practical terms:</u>

Define Voltage, $V \equiv -\int_a^b E \cdot dl = V_b - V_a$. | also: $\frac{dV}{dl} = -E$

In a wire, the electric field is uniform, hence:

$V = E(b-a) = El$ (l = length of wire)

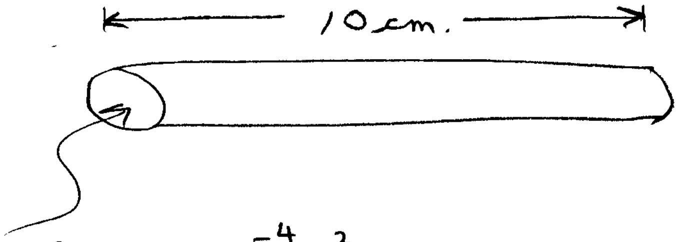
$\therefore \; \underset{\text{volts}}{V} = \left(\frac{J}{\sigma}\right) l = \frac{I}{A\sigma} l = \left(\frac{l}{A\sigma}\right) I \equiv R \underset{\text{amps}}{I}$

R: resistance, ohms (Ω)

$R = \frac{l}{A\sigma} \equiv \rho \frac{l}{A}$ ($\rho \equiv$ resistivity $= \frac{1}{\sigma}$)

See Serway p777 for typical resistances of materials.
See Serway p778 for colour coding of resistances.

Example: Ω of a Conductor (# 27.3)



Area $= 2 \times 10^{-4}\ m^2$

$\rho_{Al} = 2.82 \times 10^{-8}\ \Omega \cdot m$

$\rho_{glass} = 3.0 \times 10^{10}\ \Omega \cdot m.$

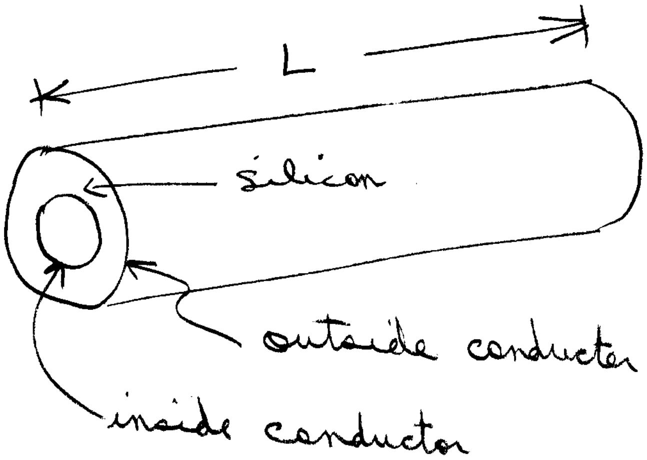
$$R_{Al} = \rho \frac{l}{A} = \frac{(2.82 \times 10^{-8}\ \Omega \cdot m)(0.100 m)}{2.00 \times 10^{-4}\ m^2}$$

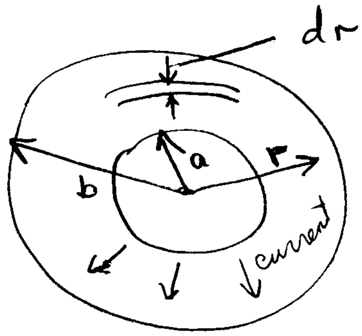
$$= 1.41 \times 10^{-5}\ \Omega$$

$$R_{glass} = \frac{3.00 \times 10^{10} \times 0.100}{2.00 \times 10^{-4}} = 1.5 \times 10^{13}\ \Omega$$

Example — Ω of a Coxial Cable (#27.5) (inside to outside)

By the book





$a = 0.5$ cm
$b = 1.75$ cm.
$L = 15.0$ cm.
$\rho_{si} = 640\,\Omega\cdot m$

$$dR = \frac{\rho\, dr}{A} = \frac{\rho\, dr}{2\pi r L}$$

$$\therefore R = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$

$$= \frac{640}{2\pi(0.15)}\ln\left(\frac{1.75}{0.5}\right)\left[\frac{\Omega\cdot m}{m}\right] = 851\,\Omega.$$

If 12 V is applied, $I = \frac{V}{R} = \frac{12}{851} = 0.0141$ A
$= 14.1$ mA

Alternate approach:

I = total current from a to b = constant at any r

$= J(r)\cdot \text{Area}(r)$

$= \sigma \underbrace{E(r)}_{= -\frac{dV}{dr}} \cdot 2\pi r L = -\frac{2\pi L}{\rho}\, r\, \frac{dV}{dr}$

$$\therefore dV = -\frac{\rho I}{2\pi L}\frac{dr}{r} \Rightarrow V_{ab} = V_b - V_a = -\frac{\rho I}{2\pi L}\ln\left(\frac{b}{a}\right)$$

Voltage at b is lower than at a, $\therefore V_{ab}$ −ve.

$$\therefore R = \frac{V_{ba}}{I} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$ as we found before.

27.3 Resistance and Temperature

Linear approximation:

$$\rho = \rho_0 [1 + \alpha (T - T_0)] \quad \text{, ie} \quad \frac{\partial \rho}{\partial T} = \alpha \rho_0$$

α — temp. coefficient

or $R = R_0 [1 + \alpha (T - T_0)]$

R_0 — reference resistance; T_0 — reference Temp

Example: Platinum Resistance Thermometer (#27.6)

used to infer T by measuring Ω changes.

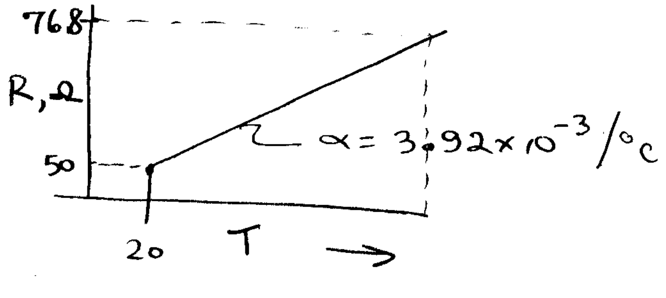
$\Omega_{20°C} = 50.0\,\Omega$ $\qquad\qquad$ $\Omega_T = 76.8\,\Omega$.

$\alpha = 3.92 \times 10^{-3}\ °C^{-1}$

$$\therefore R = R_0 [1 + \alpha (T - T_0)] = 50 [1 + 3.92 \times 10^{-3} (T - 20)]$$

$$\frac{1}{\alpha}\left[\frac{R}{R_0} - 1\right] + T_0 = T \qquad = 76.8\,\Omega$$

$$\therefore T = 20°C + \left[\frac{76.8}{50} - 1\right] \times \frac{1}{3.92 \times 10^{-3}} = 156.7\,°C$$

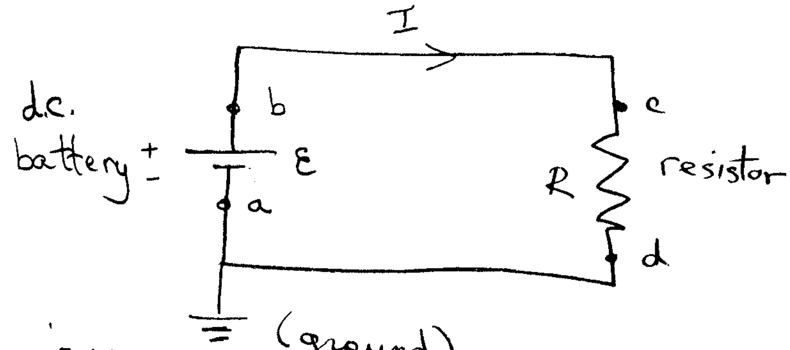


27.6 Electrical Energy and Power

(omit 27.4 and 27.5)

Consider a simple circuit:

Current flows from the battery through the resistor and back to the battery.



Assume the connecting wires bc and ad are 'ideal' (ie lossless)

Note the symbols and conventions.
$\mathcal{E}$ = Emf = electromotive force

Now: ΔU = Force × Δ distance = work

$$= q\int_a^b E\,dl$$

$$= q V_{ab}$$ = work done on moving charges through the battery potential energy rise (V_{ab})

$$\equiv qV$$

This work energy is dissipated in the resistor.
On a unit time basis:

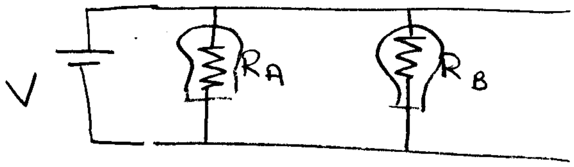
$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t} V_{ab} = IV = \text{Power}, P$$ (watts)

$$P = IV = I\cdot IR = I^2 R = \frac{V^2}{R}$$ (recall $V = IR$)

P: watts (W); I: amps (A); V: volts (V); R: ohms (Ω)

This is the power dissipated in the resistor.

Example 2 Lightbulbs (# 27.8)



Each bulb sees the same voltage.

$$R_A = 2R_B$$

$$P_A = \frac{V^2}{R_A} \qquad P_B = \frac{V^2}{R_B} = \frac{2V^2}{R_A}$$

$$\therefore \underline{P_B = 2P_A}$$

$$I = \frac{V}{R} \quad \therefore \quad I_B = \frac{V}{R_B} = \frac{2V}{R_A} = 2I_A$$

$$\therefore \underline{I_B = 2I_A}$$

Example Power in an Electric Heater (# 27.9)

$V = 110$ Volts, $R = 8\,\Omega$

$$I = \frac{110}{8} = 13.8\ A, \quad P = I^2R = (13.8)^2(8.00)$$

$$= 1520\ W$$

$$= 1.52\ kW$$

$$= \frac{V^2}{R} = IV$$

Note: If we double V, then current also doubles.
Thus Power ↑ 4x (since $P = IV$)