

Chapter 20 Heat and the 1st Law of Thermodynamics

20.1 Heat and Thermal Energy

Definitions:

calorie: amt. of heat required to raise
1 g of water from 14.5°C to 15.5°C

Calorie = 1000 calories = 1 kcal.
↑ capital "C". (appears in the food industry)

BTU = British Thermal Unit = amt. of heat
required to raise 1 lb of water from 63°F to 64°F

Joule = SI unit.

$$1 \text{ cal} = 4.186 \text{ J.}$$

Example: Losing Weight the Hard Way

Burning off a large meal by exercise.

$$\begin{aligned} 1 \text{ large meal} &\approx 2000 \text{ Calories} = 2 \times 10^6 \text{ calories} \\ &= 2 \times 10^6 \times 4.186 \text{ J/cal} = 8.37 \times 10^6 \text{ J.} \end{aligned}$$

Lifting $50 \text{ kg} \times 2 \text{ m.} \times n$ times to use $8.37 \times 10^6 \text{ J}$

implies:

$$n = \frac{8.37 \times 10^6}{\underbrace{50 \times 2 \times 9.8 \text{ m/s}^2}_{mgh}} = \underline{\underline{8.54 \times 10^3 \text{ times!}}}$$

20.2 Heat Capacity and Specific Heat

When heat, Q , is added to a mass, m , the temperature rise is governed by

$$Q = m c \Delta T$$

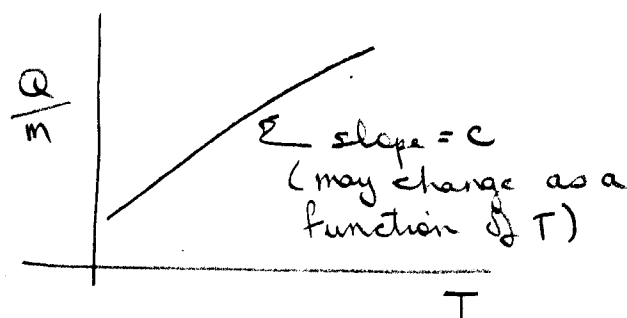
where c = heat capacity, $J/kg^\circ C$

if no work is done and no phase change occurs.

More precisely:

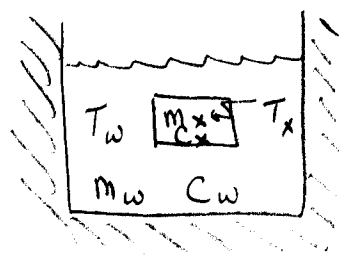
$$Q = m \int_{T_1}^{T_2} c(T) dT$$

$$\text{or } \frac{dQ}{dT} = m c(T)$$



Calorimetry

If we mix a known quantity, m_x of a substance, whose c we don't know, with a substance whose m & c are known (say water), then we can find c_x by measuring T 's before and after mixing



$$m_w c_w (T - T_w) = -m_x c_x (T - T_x)$$

energy gained by water = energy lost by x

$$\therefore c_x = \frac{m_w c_w (T - T_w)}{m_x (T_x - T)}$$

Example: Cooling a Hot Ingot

$$m_x = 0.05 \text{ kg}$$

$$m_w = 0.4 \text{ kg}$$

$$T = 224^\circ\text{C}$$

$$T_x = 200^\circ\text{C}$$

$$T_w = 20^\circ\text{C}$$

Find C_x .

$$C_x = \frac{0.4 \text{ kg} \times 4186 \text{ J/kg}^\circ\text{C} (22.4 - 20)}{0.05 \text{ kg} \times (200 - 22.4)}$$

$$= 453 \text{ J/kg}^\circ\text{C} \leftarrow \text{This is close to } C \text{ for iron.}$$

20.3 Latent Heat

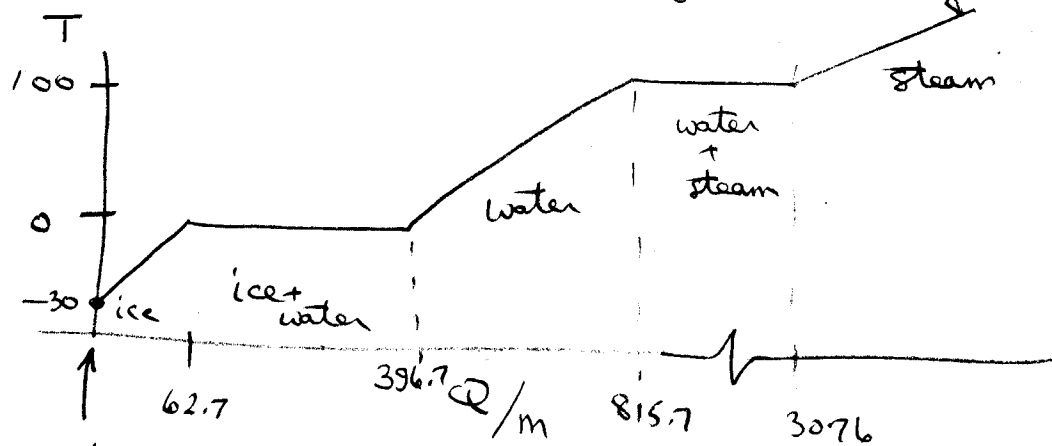
During a phase change:

$$Q = mL$$

↑ latent heat (J/kg)

L_f = latent heat of fusion (solid to liquid)

L_v = " " " " vaporization (liquid to vapour)

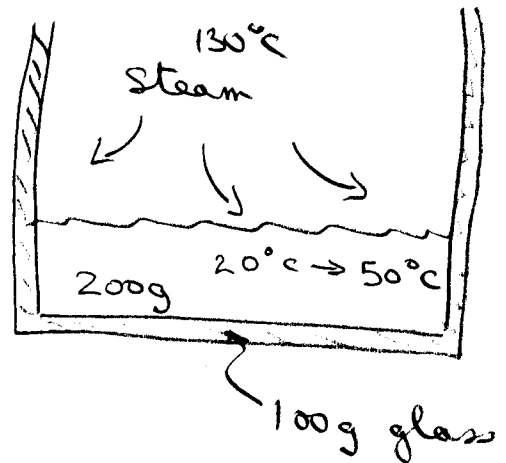


arbitrary
reference point.

$$\frac{Q}{m} = c_i \Delta T_{ice} + L_f + c_w \Delta T_w + L_v + c_s \Delta T_s$$

Example: Cooling the steam

What mass of steam is needed to warm the water and glass from $20^\circ\text{C} \rightarrow 50^\circ\text{C}$?



$$Q = \underbrace{m_s c_s \Delta T}_{\substack{\text{steam} \\ 130^\circ\text{C} \rightarrow 100^\circ\text{C}}} + \underbrace{m_s L_v}_{\substack{\text{steam} \\ \text{condensation}}} + \underbrace{m_s c_w \Delta T}_{\substack{\text{water} \\ 100^\circ\text{C} \rightarrow 50^\circ\text{C}}}$$

$$= \underbrace{m_w c_w \Delta T}_{\substack{\text{water warming} \\ 20^\circ\text{C} \rightarrow 50^\circ\text{C}}} + \underbrace{m_g c_g \Delta T}_{\substack{\text{glass warming} \\ 20^\circ\text{C} \rightarrow 50^\circ\text{C}}}$$

$$\therefore m_s \left[(2.01 \times 10^3 \text{ J/kg}^\circ\text{C})(30^\circ\text{C}) + 2.26 \times 10^6 \text{ J/kg} + 4.19 \times 10^3 \text{ J/kg}^\circ\text{C} \times 50^\circ\text{C} \right]$$

$$= 0.2 \times 4.19 \times 10^3 \text{ J/kg}^\circ\text{C} \times 30^\circ\text{C} + 0.1 \times 837 \text{ J/kg}^\circ\text{C} \times 30^\circ\text{C}$$

$$\therefore m_s = 10.9 \times 10^{-3} \text{ kg} = \underline{\underline{10.9 \text{ g}}}$$

20.4 Work and Heat in Thermodynamic Processes

State variables:

microscopic $\rho, T, P, e \dots$

any 2 fully describes a homogeneous substance in equilibrium.

macroscopic $M, V, T, P, e \dots$

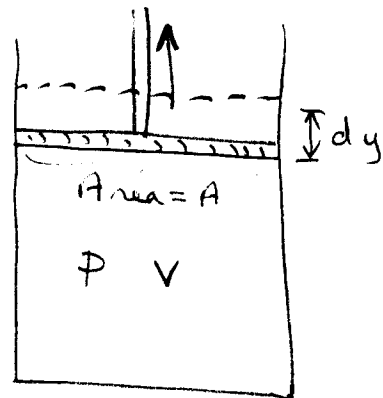
any 3 fully describes a homogeneous substance in equilibrium.

We consider a gas in thermodynamic equilibrium next.

Expand the gas quasi-statically, i.e. slowly so gas remains in equilibrium:

$$dW = F dy = PA dy = PdV$$

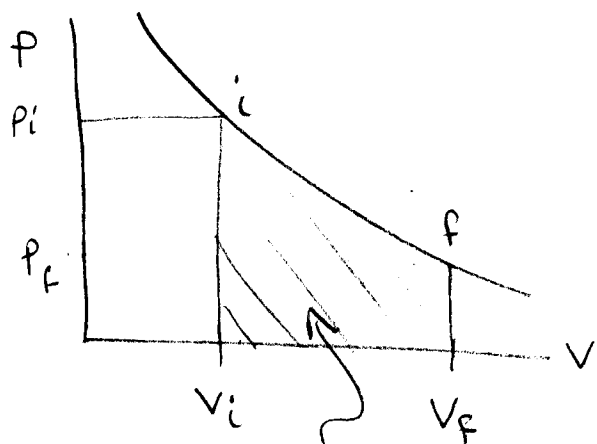
= work done by the gas on the piston.



$$\underline{\underline{dW = PdV}}$$

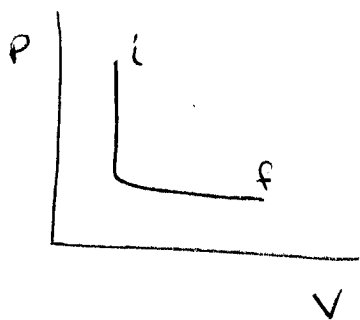
$$\therefore W = \int_{V_i}^{V_f} PdV$$

= area under curve on a PV diagram.

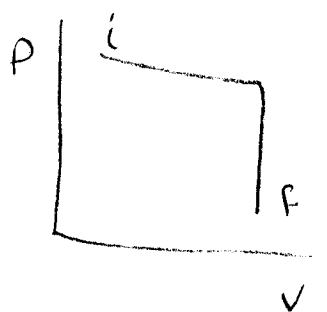


Note

The work done depends on the path.



\Leftrightarrow



↗ same initial & final states but the work done is very different.

20.5 The First Law of Thermodynamics

$$\Delta U = U_f - U_i = Q - W$$

or

$$dU = dQ - dW$$

\uparrow energy transferred to system
 \uparrow work done by the system

- U is a state function (i.e. internal energy is a condition of the state of a substance - independent of how it got there, i.e. independent of the path)
- $Q + W$ both depend on the path but $Q - W$ doesn't!
- In an isolated system, $Q = W = 0 \therefore \Delta U = 0$.
- In a non-isolated system that cycles (i.e. final state = initial state)

$$\Delta U = 0 \Rightarrow Q = W$$

Note that the area enclosed by the cycle on a PV diagram = net work done per cycle.

20.6 Some Applications of the First Law of Thermodynamics

Adiabatic Process: $Q = 0$, i.e. no thermal energy enters or leaves the system.

$$\therefore \Delta U = -W$$

Thus if a gas expands adiabatically, the system does work on its surroundings $\Rightarrow W > 0$.
 $\Rightarrow \Delta U < 0 \therefore T_{\text{gas}} \downarrow$. The converse is true.

Examples:

- expansion of hot gases in the internal combustion engine.
- liquefaction of gases in a cooling system
- compression stroke in a diesel engine.

Adiabatic free expansion; $Q = 0$
 $W = 0$ (no work done)

$\therefore \Delta U = 0 \therefore$ no change in temperature.

Isobaric Process : $P = \text{constant}$

$$W = P(V_f - V_i)$$

$$Q = 0 \text{ (usually)} \Rightarrow \Delta U = -W$$

Isovolumetric Process : $V = \text{constant}$

$$\text{Since } V = \text{constant}, W = 0$$

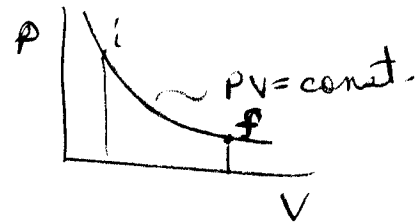
$$\therefore \Delta U = Q$$

Example: Combustion in a gasoline engine occurs quickly. All the Q is added before any ΔV occurs $\therefore \Delta U \uparrow \therefore T \uparrow$

Isothermal Process : $T = \text{constant}$

For an ideal gas, $PV = nRT$

$$\therefore PV = \text{constant} \Rightarrow$$



$$\Rightarrow \Delta U = 0$$

Since work is done by the system, some Q must be added to maintain T constant.

$$W = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} \frac{nRT}{V} dV = nRT \ln V \Big|_{V_i}^{V_f}$$

$$= nRT \ln(V_f/V_i)$$

$$= Q \text{ added.}$$

Example: Work Done During an Isothermal Expansion

1 mole of an ideal gas at 0°C , $V_i = 3\text{ l}$, $V_f = 10\text{ l}$.

Find W .

$$\begin{aligned} W &= nRT \ln(V_f/V_i) \\ &= 1\text{ mol} \times 8.31\text{ J/(mol}\cdot\text{K)} \times 273\text{ K} \times \ln(10/3) \\ &= 2.7 \times 10^3\text{ J} = Q_{\text{added}}. \end{aligned}$$

Boiling

Consider vaporizing m kg of liquid at constant pressure.

$$W = \int_{V_l}^{V_v} P \, dV = P \int dV = P(V_v - V_l)$$

$v = \text{vapor}$
 $l = \text{liquid}$

$$Q_{\text{added}} = mL_v$$

$$\therefore \Delta U = Q - W = mL_v - P(V_v - V_l)$$

Example: Boiling Water

1 gm at $P = P_{\text{atm}}$, $V_l = 1\text{ cm}^3$. Find ΔU .

$$Q = mL_v = (1.0 \times 10^{-3}\text{ kg}) \times (2.26 \times 10^6\text{ J/kg}) = 2260\text{ J}$$

$$\begin{aligned} W &= P(V_v - V_l) = (1.013 \times 10^5\text{ N/m}^2) [1671 - 1.00] \times 10^{-6}\text{ m}^3 \\ &= 169\text{ J} \end{aligned}$$

$$\therefore \Delta U = Q - W = 2260\text{ J} - 169\text{ J} = 2.09\text{ kJ}$$

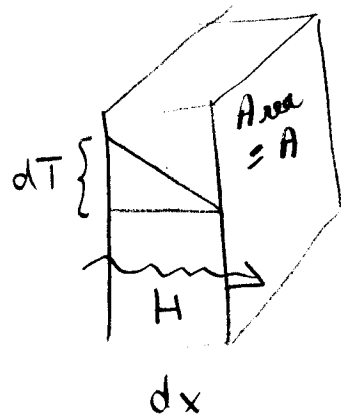
ie 93% of the heat added goes to ΔU , 7% goes to work.

20.7 Heat Transfer

Heat Conduction

$$\frac{Q}{\Delta t} \equiv H = \text{heat transfer rate} = -kA \frac{dT}{dx}$$

temp. gradient



thermal conductivity

$$k [] \frac{W}{m \cdot ^\circ C}$$

In steady state, no heat is accumulating anywhere, thus, in a slab, $H = \text{constant}$ for all x .

$$H = -kA \frac{dT}{dx}$$

$$\therefore dT = -\frac{1}{kA} H dx$$

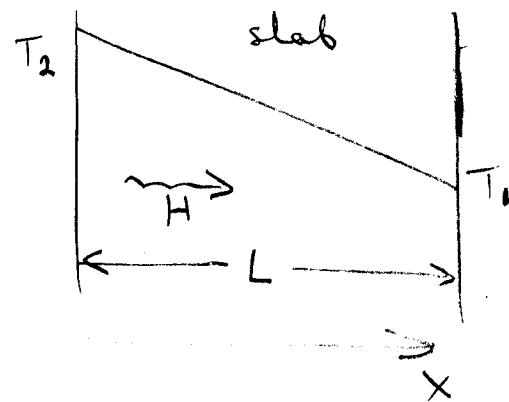
$$\therefore \int_{T_2}^{T_1} dT = -\frac{1}{kA} H \int_{x_2}^{x_1} dx$$

$$\therefore T_1 - T_2 = -\frac{1}{kA} H (x_1 - x_2) = -\frac{1}{kA} H L$$

$$\therefore H = \frac{kA (T_2 - T_1)}{L}$$

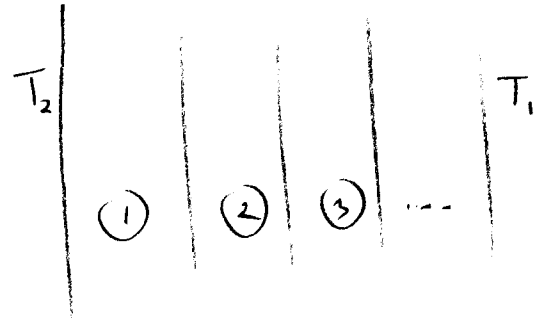
In general $T(x) = \frac{1}{kA} H x + T_2$

ie linear variation in a slab.



For slabs in series:

$$\int_{T_2}^{T_1} dT = -\frac{1}{A} H \int_{x_2}^{x_1} \frac{1}{k(x)} dx$$



$$= -\frac{H}{A} \int_{\text{①}} \frac{dx}{k_1} + \int_{\text{②}} \frac{dx}{k_2} + \dots$$

$$= -\frac{H}{A} \left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \dots \right) = -\frac{H}{A} \sum_i \frac{L_i}{k_i}$$

$$= T_1 - T_2$$

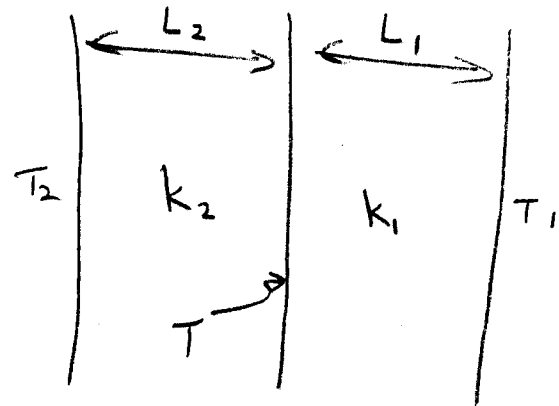
$$\therefore H = \frac{A(T_2 - T_1)}{\sum_i (L_i/k_i)}$$

Example: Heat Transfer Through Two Slabs

Find T at interface & H .

$$H_1 = k_1 A \frac{(T - T_1)}{L_1}$$

$$H_2 = k_2 A \frac{(T_2 - T)}{L_2}$$



But $H_1 = H_2 \therefore \frac{k_1 (T - T_1)}{L_1} = \frac{k_2 (T_2 - T)}{L_2}$

Solving for T :

$$T = \frac{k_1 T_1 L_2 + k_2 T_2 L_1}{(L_1 k_1 + L_2 k_2)}$$

$$+ H = k_1 A \frac{(T - T_1)}{L_1} \left(= k_2 A \frac{(T_2 - T)}{L_2} \right)$$

See Serway table 20.3 for typical values of k .

Home Insulation

We have shown that

$$H = \frac{A(T_2 - T_1)}{\sum_i L_i/k_i}$$

In engineering practice, $L/k = R$ [\equiv] $\frac{\text{ft}^2 \cdot \text{h}}{\text{BTU}}$

$$\therefore H = \frac{A(T_2 - T_1)}{\sum_i R_i}$$

See Serway table 20.4 for typical R values of building materials.

eg: 3 1/2" fiberglass batting $R = 10.9$

Example: R value of a Typical Wall

$$R_1 \text{ (outside air film)} = 0.17 \text{ ft}^2 \cdot \text{h}/\text{BTU}$$

$$R_2 \text{ (brick)} = 4.00$$

$$R_3 \text{ (sheathing)} = 1.32$$

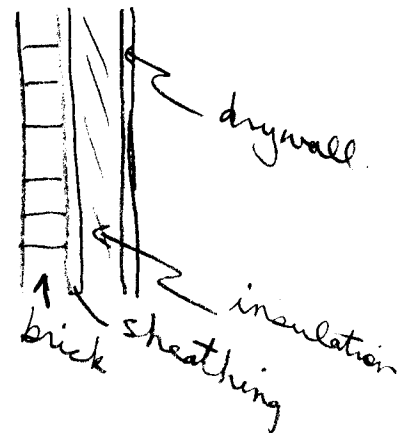
$$R_4 \text{ (insulation)} = 10.9$$

$$R_5 \text{ (drywall)} = 0.45$$

$$R_6 \text{ (inside air film)} = 0.17$$

R_{total}

$$17.01 \text{ ft}^2 \cdot \text{h}/\text{BTU}$$

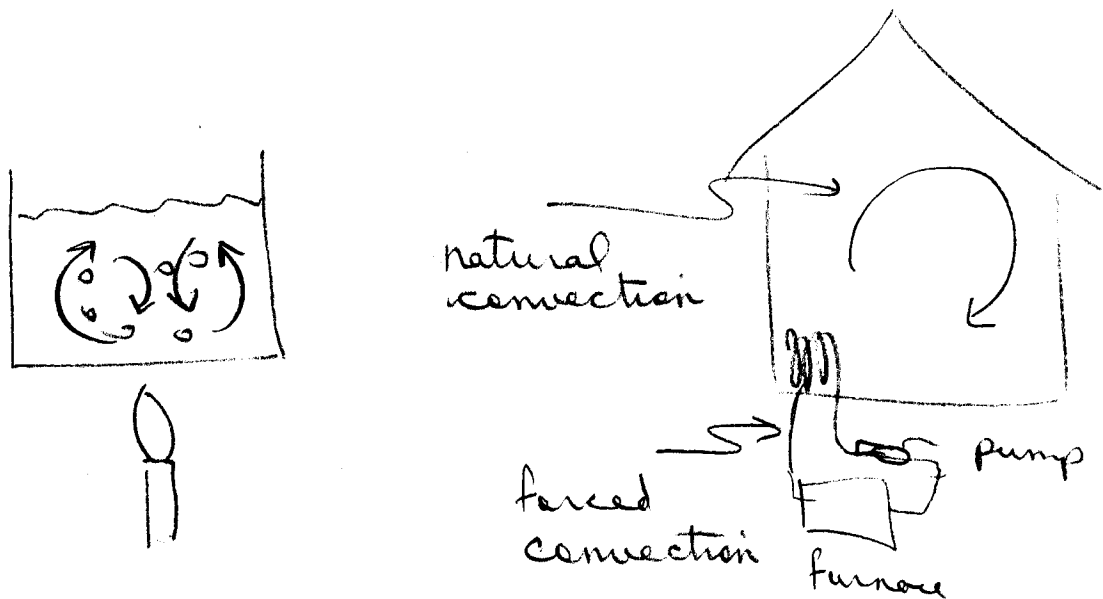


Convection

Thermal energy transferred by movement of a heated substance \equiv convection.

If the substance (usually a fluid) is pumped, we call this forced convection.

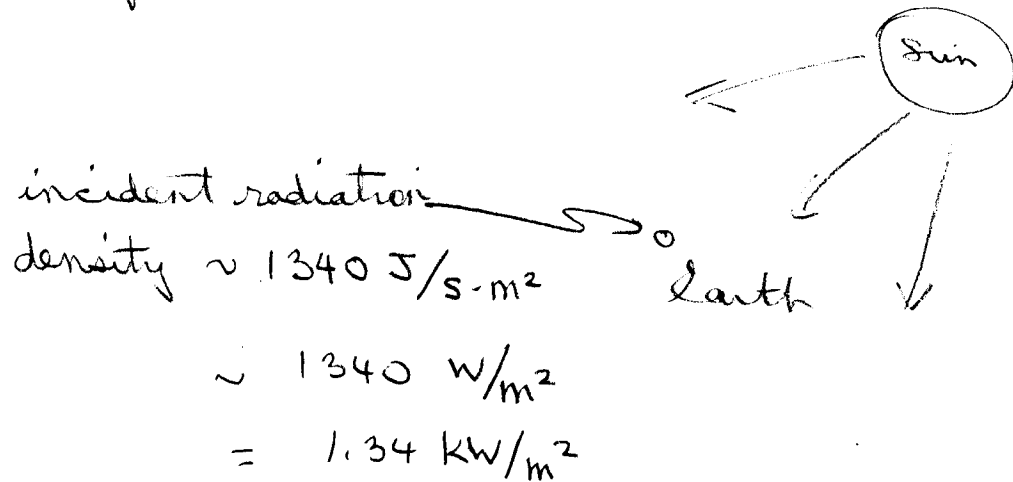
If the fluid moves by virtue of density differences (caused by temperature differences), we call this natural convection.



This subject is beyond the scope of this course.

Radiation

- Electromagnetic Waves



That is more than we need - if only we could capture it efficiently, economically, etc.

- All objects emit radiant heat.

- Stefan's Law : $P \text{ (watts)} = \sigma A e T^4$

$$\sigma = 5.6696 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$

A = surface area, m^2

e = emissivity (dimensionless)

T = temperature, K .

An object receives (absorbs) radiation + emits radiation.

If the temperature of the surroundings is T_0 ,

then $P_{\text{net body}} = \sigma A e (T^4 - T_0^4)$ and the object heats up (if $T < T_0$) or cools down (if $T > T_0$) until $T = T_0$ (thermal equilibrium.)

$e = 1$ ideal absorber (and radiator)
This is called a black body.

$e = 0$ perfect reflector

Example: Who Turned Down the Thermostat

Heat loss through bare skin?

$A \sim 1.5 \text{ m}^2$ per person

$T_0 = 20^\circ\text{C}$

$T_{\text{person}} = T = 37^\circ\text{C}$

$e = 0.9$

$$P_{\text{net}} = \sigma A e (T^4 - T_0^4) = (5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4) (1.5 \text{ m}^2) \times (0.9) [(37+273)^4 - (20+273)^4] = 140 \text{ J/s}$$