ENGINEERING 2O3

DAY CLASS — Dr. Wm. Garland

DURATION: 50 minutes

McMASTER UNIVERSITY MIDTERM EXAMINATION — February 24, 1999

Special Instructions:

1. Closed Book. All calculators and up to 3 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions. Place your answers on the exam sheets; use additional pages if necessary.
3. The value of each part is as indicated. TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 3 PAGES AND 3 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [30 marks total] An electric car is designed to run off a bank of 12-V batteries (connected in parallel) with total energy storage of 2.0×10^7 J.
   a. If the electric motor draws 8.0 kW, what is the current delivered to the motor?
   b. If the electric motor draws 8.0 kW, as the car moves at a steady speed of 20 m/s, how far will the car travel before it is "out of juice"?

Solution

(a) Since $P = VI$

$$I = \frac{P}{V} = \frac{8.0 \times 10^3 \text{ W}}{12 \text{ V}} = 667 \text{ A}$$

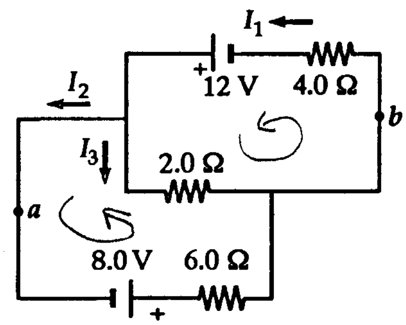
(b) We find the time the car runs from $P = \frac{U}{t}$

$$t = \frac{U}{P} = \left(\frac{2.0 \times 10^7 \text{ J}}{8.0 \times 10^3 \text{ W}}\right)\left(\frac{1 \text{ W} \cdot \text{s}}{\text{J}}\right) = 2.5 \times 10^3 \text{ s}$$

So it moves a distance of

$$x = vt = (20 \text{ m/s})(2.5 \times 10^3 \text{ s}) = 50 \text{ km}$$

2. [30 marks] For the circuit shown in the figure, calculate
   a. the current in the 2.0-Ω resistor and
   b. the potential difference between points *a* and *b*.
   [Hint: start by picking current directions and loop directions, as shown.]



Solution Arbitrarily choose current directions as labeled in the figure to the right.

(a) From the junction point rule, we have

$$I_1 = I_2 + I_3 \qquad (1)$$

Traversing the top loop counterclockwise gives

$$(12\text{ V}) - (2.0\ \Omega)I_3 - (4.0\ \Omega)I_1 = 0 \qquad (2)$$

Traversing the bottom loop counterclockwise,

$$8.0\text{ V} - (6.0\ \Omega)I_2 + (2.0\ \Omega)\ I_3 = 0 \qquad (3)$$

From Equation (2), $I_1 = (3.0\text{ A}) - \dfrac{I_3}{2.0}$

From Equation (3), $I_2 = \dfrac{(4.0\text{ A}) + I_3}{3.0}$

Substituting these values into Equation (1), we find that $I_3 = 0.909$ A.

Therefore, the current in the 2.0-Ω resistor is 0.91 A ◊

(b) $V_a - (0.909\text{ A})(2.0\ \Omega) = V_b$,

Therefore, $V_a - V_b = 1.8\text{ V}$, with $V_a > V_b$ ◊

3. [40 marks] Consider a series RLC circuit having the following circuit parameters: R = 200 Ω, L = 663 mH, and C = 26.5 μF. The applied voltage has an amplitude of 50.0 V and a frequency of 60.0 Hz. Find the following amplitudes:
   a. The current i, including its phase constant ϕ relative to the applied voltage.
   b. The voltage V_R across the resistor and its phase relative to the current.
   c. The voltage V_C across the capacitor and its phase relative to the current.
   d. The voltage V_L across the inductor and its phase relative to the current.

Solution We identify that

$$R = 200\ \Omega,\ L = 663\ \text{mH},\ C = 26.5\ \mu\text{F},\ \omega = 377\ \text{rad/s},\ \text{and}\ V_{\max} = 50\ \text{V}$$

So

$$\omega L = 250\ \Omega, \quad \text{and} \quad \left(\frac{1}{\omega C}\right) = 100\ \Omega$$

The impedance is

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{(200\ \Omega)^2 + (250\ \Omega - 100\ \Omega)^2} = 250\ \Omega$$

(a) $I = \frac{V}{Z} = \frac{50\ \text{V}}{250\ \Omega} = 0.200\ \text{A}$ ◊

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = 36.8°$ ◊ with V leading I

(b) $V_R = IR = 40.0\ \text{V}$ at $\phi = 0°$ ◊

(c) $V_C = IX_C = (0.200\ \text{A})(100\ \Omega) = 20.0\ \text{V}$ at $\phi = -90.0°$ ◊

(d) $V_L = IX_L = (0.200\ \text{A})(250\ \Omega) = 50.0\ \text{V}$ at $\phi = 90.0°$ ◊