Name: _____ ID: _____

ENGINEERING 2C03

DAY CLASS — Dr. Wm. Garland

DURATION: 50 minutes

McMASTER UNIVERSITY MIDTERM EXAMINATION — February 27, 2001

Special Instructions:

1. Closed Book. All calculators and up to 6 single sided 8 ½" by 11" crib sheets are permitted.
2. Do all questions. Place your answers on the exam sheets; use additional pages if necessary.
3. The value of each part is as indicated. TOTAL Value: 100 marks

THIS EXAMINATION PAPER INCLUDES 3 PAGES AND 3 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

1. [30 marks total] Suppose that the current through a conductor decreases exponentially with time according to

$$I(t) = I_0 e^{-t/\tau}$$

where I_0 is the initial current (at t=0), and τ is the time constant having dimensions of time. Consider a fixed observation point within the conductor. How much charge passes this point between t=0 and t=τ?

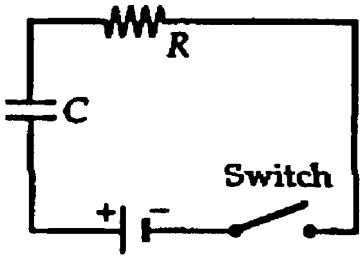
Sol'n:

$$I = \frac{dQ}{dt} \Rightarrow Q = \int_0^{\tau} I\,dt = I_0 \int_0^{\tau} e^{-t/\tau}\,dt$$

$$= -\tau e^{-t/\tau}\Big|_0^{\tau} = \tau(1 - e^{-1})$$

$$= 0.6321\, I_0 \tau$$

2. [30 marks] Consider a series RC circuit, as shown in the figure, for which R = 1.00 MΩ, C = 5.00 μF, and ε = 30.0 V.
   a. Find the time constant of the circuit.
   b. Find the maximum charge on the capacitor after the switch is closed.
   c. If the switch is closed at t = 0, find the current in the resistor 10.0 s later.



Soln:

We have $\mathcal{E} - q/C - IR = 0$, $I = \frac{dq}{dt}$

$\Rightarrow \frac{dq}{dt} = \mathcal{E}/R - q/RC \Rightarrow q = \mathcal{E}C(1 - e^{-t/\tau})$

& $I = \mathcal{E}/R\, e^{-t/RC}$; $\tau = RC$

∴ a) $\tau = RC = 1 \times 10^{6} \times 5 \times 10^{-6} = 5.00$ s.

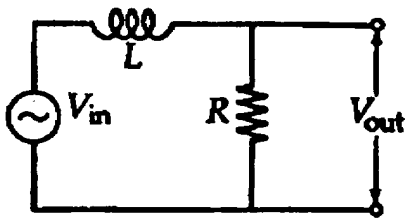
b) at $t = \infty$, $q = \mathcal{E}C = 5 \times 10^{-6} \times 30$

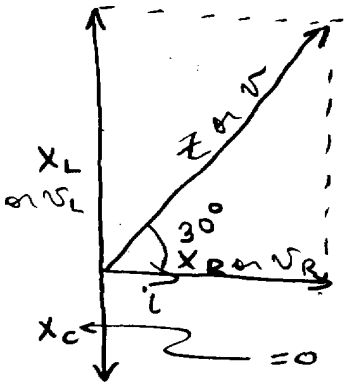
$= 150\ \mu C$

c) $I = \frac{30}{1 \times 10^{6}}\, e^{-10/5} = 4.06 \times 10^{-6}$ A

$= 4.06\ \mu A$

3. [40 marks] Consider the phase-shifter circuit shown in the figure. The input voltage is described by the expression v = (10 V) sin 200t (in SI units). Assuming that L = 500 mH.
   a. Find the value of R such that the output voltage v_{out} lags behind the input voltage by 30°.
   b. Find the amplitude of the output voltage.





a) $\tan 30^\circ = \frac{X_L - X_C}{X_R} = \frac{X_L}{X_R}$

$\therefore R = \frac{X_L}{\tan 30^\circ} = \frac{\omega L}{\tan 30^\circ} = \frac{200 \text{ rad/s} \times 0.5 \text{ H}}{\tan 30^\circ}$

$= 173\ \Omega$

b) $\frac{V_{out}}{V_{in}} = \frac{iR}{iZ} = \frac{R}{\sqrt{R^2 + X_L^2}}$

$\therefore V_{out} = \frac{10 \times 173}{\sqrt{173^2 + 100^2}} = 8.66 \text{ V}$