Transient Diffusion in a solid

$$\rho c \frac{\partial T}{\partial t} = q''' + \nabla \cdot k \nabla T$$

ρ = density, c = heat capacity, q''' = heat source per unit volume

$$\rightarrow = k \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) \text{ in 1-D with constant } k$$

$$\frac{T_i^{t+\Delta t} - T_i^t}{\Delta t}$$

$$k\left[\frac{\partial T}{\partial x}\Big|_{i+\frac{1}{2}} - \frac{\partial T}{\partial x}\Big|_{i-\frac{1}{2}}\right] \Big/ \Delta x$$

$$= k\left[\frac{(T_{i+1} - T_i) - (T_i - T_{i-1})}{\Delta x}\right] \Big/ \Delta x$$

$$= k\frac{(T_{i+1} - 2T_i - T_{i-1})}{\Delta x^2}$$

$$\therefore \rho c \frac{(T_i^{t+\Delta t} - T_i^t)}{\Delta t} = q''' + \frac{k}{\Delta x^2}\left(T_{i+1}^t - 2T_i^t + T_{i-1}^t\right)$$

$$\therefore T_i^{t+\Delta t} = T_i^t + \frac{\Delta t}{\rho c}\left[q''' + \frac{k}{\Delta x^2}\left(T_{i+1}^t - 2T_i^t + T_{i-1}^t\right)\right]$$