Nuclear Theory - Course 227

NEUTRON CROSS SECTIONS, NEUTRON DENSITY AND NEUTRON FLUX

Neutron Cross Sections

Let us have a look at the various reactions a neutron can undergo with a U-235 nucleus:

As mentioned in lesson 227.00-2:

1. If the neutron energy is greater than 0.1 MeV, inelastic scattering may occur. If the neutron energy is less than this, there is no chance of this reaction happening.
2. The neutron may just bounce off (elastic scattering), and this can happen at all neutron energies.
3. The neutron may be captured (radiative capture).
4. The neutron may cause fission.

Radiative capture and fission are much more likely for slow neutrons than for fast neutrons, and fission is always more probable than radiative capture.

Thus we are always comparing the chances in favour of the various reactions taking place. It is the probability of a certain reaction occurring that is important. Some reactions are more probable with some nuclei than with others or more probable with some neutron energies than with others. Because these reactions are concerned with a neutron striking a target, namely a nucleus, the probability that a certain reaction will occur is measured in terms of an area, called the *Neutron Cross-Section*.

To understand this cross-section better, imagine the neutrons as being bullets shot at the target in Figure 1, instead of at a nucleus. When the neutron misses the target altogether, no reaction takes place. The areas of the various rings on the target represent the chance of various reactions occurring. Thus the area, d, of the complete disc, being the easiest to hit, represents the probability of the easiest reaction occurring. The area to the outside of the single-hatched ring, c, represents the probability of the next easiest reaction occurring. Area b represents the probability of the third easiest reaction occurring and area a, of the bull's eye, the probability of the most difficult reaction occurring, since the bull's eye is the most difficult to hit. The areas of these rings can be such that the probability of an area being hit by a bullet is equal

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to the probability of a reaction occurring between the neutron and the nucleus. The area of the ring is, then, the cross-section for that particular reaction. Because these cross-sections apply specifically to individual nuclei, they are known as *microscopic* cross-sections.

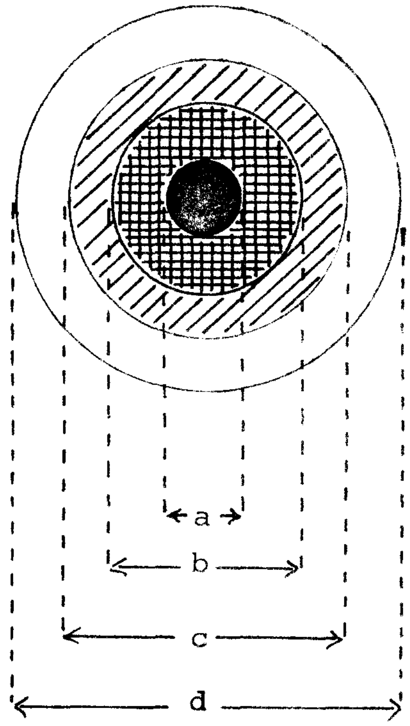


Figure 1

Needless to say the ring areas are extremely small, being of the order of 10^{-24}, 10^{-23} or 10^{-22} square centimeters. A special unit, called the *barn*, is therefore used to describe these cross-sections.

$$1 \text{ barn} = 10^{-24} \text{ cm}^2$$

The barn is of the same sort of size as the physical target area (πr^2) presented by a medium sized nucleus.

If a reaction has a large cross section, say 100 b, it will occur much more frequently than one that has a small cross-section, say 0.1 b. In fact, it is exactly 1000 times as likely.

As was pointed out earlier, when there are a number of possible reactions with a given nucleus, each one would have its own cross-section. The Greek letter σ (sigma) is used as the symbol for the microscopic cross-section, and so:

σ_f = fission cross section

σ_a = absorption cross section

$\sigma_{n,\gamma}$ = radiative capture cross section

σ_i = inelastic scattering cross section

σ_s = elastic scattering cross section

σ_a is usually the radiative capture cross section, ie, $\sigma_{n,\gamma}$. Only in those few cases where fission is also possible, (ie, $\sigma_f \neq 0$), σ_a would include σ_f and $\sigma_{n,\gamma}$ since a neutron is absorbed in both cases;

ie, $$\sigma_a = \sigma_f + \sigma_{n,\gamma},$$

since both fission and radiative capture involve a complete absorption and loss of the neutron. So, to repeat, for nuclides with $\sigma_f = 0$, σ_a is merely $\sigma_{n,\gamma}$.

Cross sections depend very much on the neutron energy. Generally speaking, they are a lot larger at low energies than at high energies. For example, the fission cross-section σ_f for U-235 for neutrons of thermal energy is 580 b, whereas it is only just over 1 b at MeV. In other words, fission of U-235 is about 500 times as likely for thermal neutrons than for fast neutrons. This very nicely illustrates what the moderator does for us.

For your interest, Table I lists the thermal neutron cross-sections of fuel nuclei. It might be quite instructive to have a look at these numbers and see what we can make of them.

In the table σ_a is shown as $\sigma_f + \sigma_{n,\gamma}$, since both processes involve a complete absorption of the neutron.

TABLE I

Thermal Neutron Cross Sections of Fuel Atoms (in Barns)
(taken from Atomic Energy Review (IAEA), 1969, Vol 7, No 4, p.3)

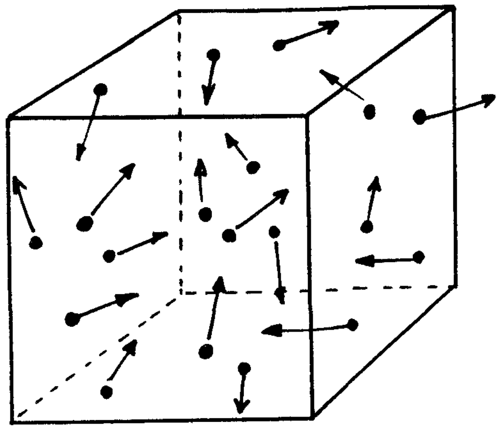
	σ_f	$\sigma_{n,\gamma}$	σ_a	σ_s	ν
U-233	530.6	47.0	577.6	10.7	2.487
U-235	580.2	98.3	678.5	17.6	2.430
U-238	0	2.71	2.71	∿ 10	0
Nat. U	4.18	3.40	7.58	∿ 10	
Pu-239	741.6	271.3	1012.9	8.5	2.890
Pu-241	1007.3	368.1	1375.4	12.0	2.934

Only 86% of the thermal neutrons absorbed by U-235 cause fission. You can see that this is just the fraction σ_f/σ_a. Note also that U-233 gives the greatest percentage of fission per neutron absorbed (σ_f/σ_a = 92%); this is a very desirable aspect of U-233, and for this reason it may well be used in future reactors.

The values for natural uranium were obtained by using 99.3% of the U-238 values and 0.7% of the U-235 values. Looking at the table, you can see that for natural uranium σ_f = 4.18 b, $\sigma_{n,\gamma}$ = 3.40 b and hence σ_a = 7.58 b. This means that for every, say, 758 thermal neutrons absorbed in natural uranium, 418 will cause fission. Since these fissions can only occur in U-235, we will get ν = 2.43 new neutrons produced per fission. The 418 fissions will therefore generate 418 x 2.43 = 1016 new neutrons. This means that for every thermal neutron absorbed in natural uranium fuel, we will on average get back 1016/758 = 1.34 new ones.

In our reactors this is a sufficient number because we have relatively few neutron losses in reactor materials (in other words, the absorption cross-sections of the reactor materials we use are small enough). However, the U.S. reactors use a light water moderator. Light water has an absorption cross-section that is almost 700 times greater than that of heavy water. As a result, the light water absorbs so many neutrons that 1.34 new neutrons for every neutron absorbed in the fuel are not enough. They therefore use *enriched* fuel, ie, the U-235 concentration is greater than the naturally occuring one of 0.72%. You might like to work out for yourself (using the values given in Table I) what difference an enrichment of 2% U-235 makes (2% enrichment means 2% U-235 and 98% U-238).

Now that we have described what cross sections are, let us take this discussion a little further. Imagine 1 cm^3 cube of a certain kind of material, and let this cube contain n thermal neutrons. These n neutrons are all zipping around inside the



cube with velocity v and they will make collisions with the nuclei sitting there. We will assume that there are N nuclei in the 1 cm^3 cube, and that their absorption (n,γ) cross-section is σ_a. It turns out that the number of neutrons interacting (ie, being absorbed) per second is given by:

$$R = nv.N\sigma_a \qquad (1)$$

R is called the *reaction rate*. Intuitively you can see that the expression for R seems reasonable, because:

(a) The larger n, the more neutrons will make collisions

(b) The larger their velocity, the more nuclei they will get to hit in a certain time,

(c) The larger the number of nuclei present (N) the more will be hit, and

(d) The larger the cross-section, the greater is the probability of getting a hit.

This result is quite general, and if we were to use σ_f instead of σ_a in the expression, R would be the number of fissions per second. The quantities N and σ are both characteristic of the so-called target material, and therefore they are often combined to form the

$$\textit{macroscopic cross section } \Sigma = N\sigma \qquad (2)$$

Σ is the capital σ, and note the spelling m*a*croscopic instead of m*i*roscopic.

The units of Σ will be cm^{-1}. For example, let us work out Σ_a for natural uranium. N is 0.048×10^{24} nuclei/cm^3 and from Table 1 σ_a is seen to be 7.58 barns.

$$\therefore \Sigma_a = N\sigma$$

$$= 0.048 \times 10^{24} \left(\frac{1}{cm^3}\right) \times 7.58 \times 10^{-24} (cm^2)$$

$$= 0.36 \ cm^{-1}$$

What does this mean? Well, please take my word for it that $1/\Sigma_a$, which is a distance, is the average distance a neutron will travel before being absorbed in the material. That is, thermal neutrons zipping around in natural uranium will travel an average distance of 1/0.36 - 2.8 cm before they are absorbed.

Appendix B gives the values of Σ_a for all of the elements and for light and heavy water. The cross-sections apply to thermal neutrons only. This table has been included for interest's sake only, but it does bring out which materials have high neutron capture cross-sections and which don't.

To return now to equation (1), we can write it as

$$R = nv.\Sigma \qquad (3)$$

n is the number of neutrons per cm^3. We call this the *neutron density*, for rather obvious reasons.

nv is called the *neutron flux*. It is the total distance travelled by all the n neutrons in 1 cm^3 in one second, since each of them will cover a distance v. The Greek letter ϕ (phi) is always used for neutron flux. Its units are

$$\frac{\text{neutron} \quad \text{cm}}{\text{cm}^3 \quad \text{s}}$$

It represents the total neutron tracklength per unit volume per unit time. We therefore end up with

$$R = \phi\Sigma$$

To see what sort of use these ideas have, let us look at an operating reactor that has an average thermal neutron density of 100 million, ie, $n = 10^8 cm^{-3}$. This is a typical figure. The speed of thermal neutrons is still quite high, it is in fact 2.2 km/s, that is $2.2 \times 10^5 cm.s^{-1}$ (or 5000 m.p.h., if you like to look at it that way). Therefore, this reactor has an average neutron flux

$$nv = \phi = 2.2 \times 10^{13} \text{ n.cm}^{-2}\text{s}^{-1}$$

If the reactor uses natural uranium, the the absorption rate per cm^3 of fuel is

$$\phi\Sigma_a = 2.2 \times 10^{13} \times 0.36 = 7 \times 10^{12} \text{ s}^{-1}$$

If the reactor contains 10^6 cm^3 of fuel (ie, 1 m^3) then there will be

$$7 \times 10^{12} \times 10^6 = 7 \times 10^{18} \text{ neutron captures per second.}$$

Going back to Table I, you can see that 4.18 in every 7.58 neutrons captured will cause fission. We will then have

$$\frac{4.18}{7.58} \times 7 \times 10^{18} = 3.8 \times 10^{18} \text{ fissions per second.}$$

We saw earlier that 3.1×10^{10} fissions per second will produce 1 watt, therefore in this case the reactor is producing

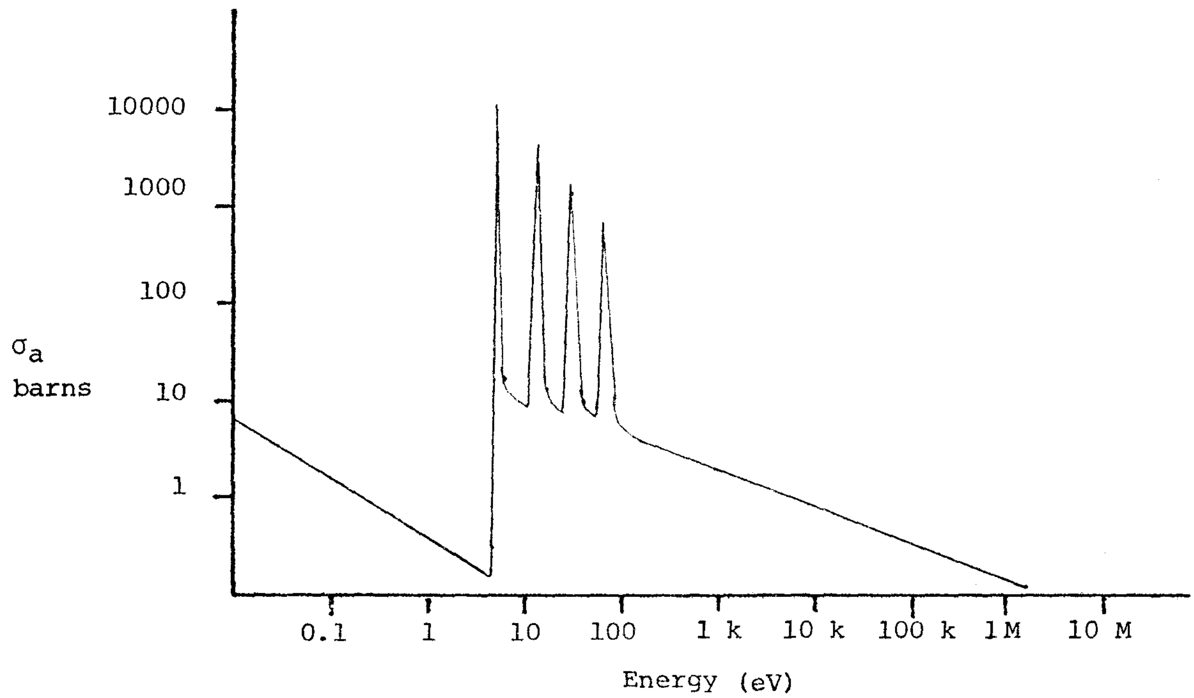
$$\frac{3.8 \times 10^{18}}{3.1 \times 10^{10}} \text{ watts} = 123 \times 10^6 \text{ W} = \underline{123 \text{ MW}} \text{ (thermal)}$$

Chart of the Nuclides

We have now covered all the material necessary to use the chart of the Nuclides which is included as Appendix C. For those self-studying this course, a few minutes spent studying the explanation of the chart will be time well spent.

Variation in Cross Sections

As mentioned, neutron cross sections are highly energy dependant. The variation in cross section is not a simple function of neutron energy. Figure 2 shows the variation of the absorption cross section of U-238 with energy. Of particular interest here are the pronounced peaks between ∿5 eV and ∿1 keV. These are called resonance absorption peaks and the corresponding energies resonance energy. The cross sections are so high in these regions that a large portion of the neutrons at these energies will be absorbed.



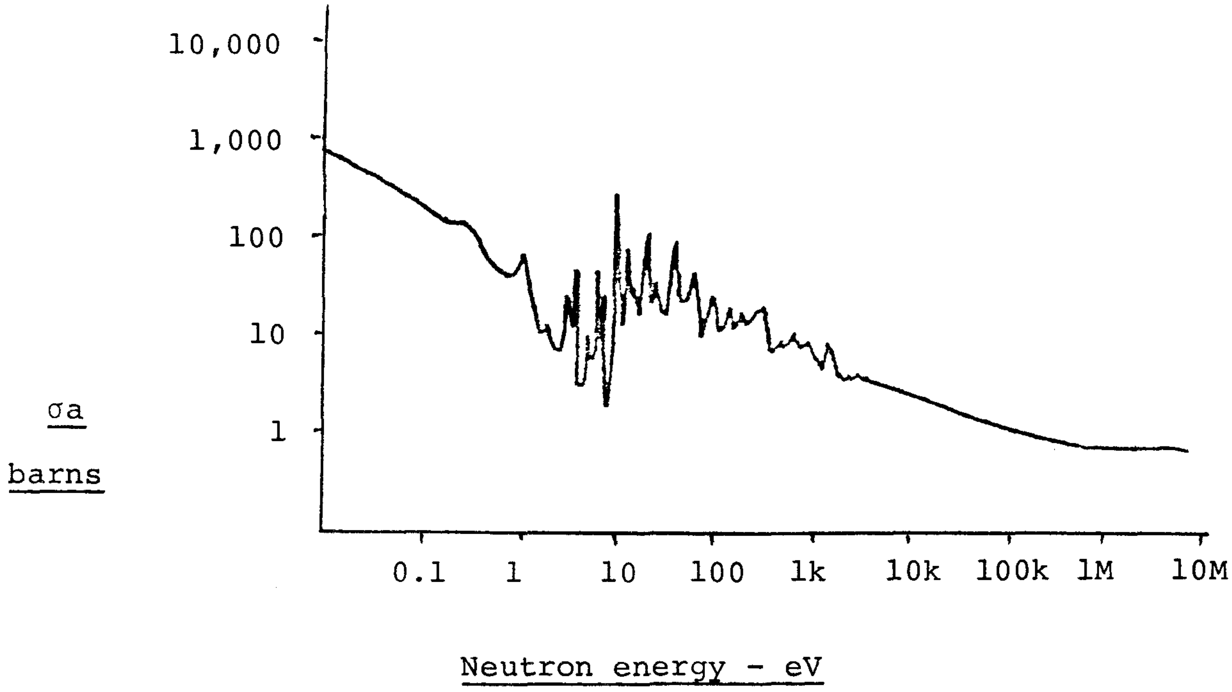
Variation of the absorption cross section of U-238 with neutron energy.

Figure 2

All cross sections have some energy dependance. At low energies most cross sections are inversely proportional to the neutron velocity, ie,

$$\sigma \propto \frac{1}{v} \text{ or } \frac{1}{\sqrt{E}}$$

Variations from this normal behavior will be covered when they have an effect on overall behavior of the reactor.



Variation of the absorption cross section of U-235 with neutron energy.

Figure 3

ASSIGNMENT

1. Explain what a microscopic neutron cross section is.
2. If 100 thermal neutrons were absorbed by natural uranium, how many fast neutrons would be produced? What is the significance of your answer?
3. Using the Chart of the Nuclides, trace the radioactive decay of U-238 to a stable nuclide.
4. If a thermal neutron interacts with a U-235 nucleus, calculate the probability that the interaction will be a scattering reaction.

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