

## Nuclear Theory - Course 227

MEANING OF CRITICALITY, MULTIPLICATION FACTOR,  
REACTIVITY AND NEUTRON LIFETIME

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In the previous lessons, we discussed a reactor operating at steady power, in which there is an exact balance between the neutrons lost and neutrons produced through fission. The neutron population remained constant, the neutron flux remained constant and the chain reaction was just being maintained.

We will now turn our attention to changes in power and in neutron density, which may occur due to a variety of reasons.

Criticality and Neutron Multiplication

In a critical chain reaction, the number of fissions in each succeeding generation of neutrons remains constant. There is no MULTIPLICATION of neutrons and the power remains constant.

Suppose that neutron losses, by non-fission capture or leakage, are reduced somehow or other. More neutrons are then available for fission and the number of fissions occurring in any one generation will be greater than in the previous generation. Thus more neutrons are produced, in any one generation, than in the previous generation. There is, therefore, a multiplication of neutrons.

Fig. 1 illustrates an extreme case of such neutron multiplication. The neutron losses have been reduced so much that two neutrons, from each fission, become available to cause further fissions. It shows the number of neutrons in succeeding generations, that would result from one original neutron. The just critical case is shown immediately underneath, for comparison. It is obvious from the figure that there is a doubling of the neutron population, and therefore a doubling of power, every generation. Since the time between one generation and the next is only about one-thousandth of a second, this means that the power would increase to 1000 times its value in one-hundredth of a second. This is obviously an extreme case, but it serves to illustrate the principle of neutron multiplication.

The ratio of the number of neutrons, available for fission, in one generation, to the number available in the previous generation is called the MULTIPLICATION FACTOR ( $k$ ). Hence

$$k = \frac{\text{Number of neutrons causing fission in the 2nd generation}}{\text{Number of neutrons causing fission in the 1st generation}}$$

In the case illustrated,  $k = 2$

The neutron multiplication factor is also the number of neutrons from each fission, which are available for further fission.

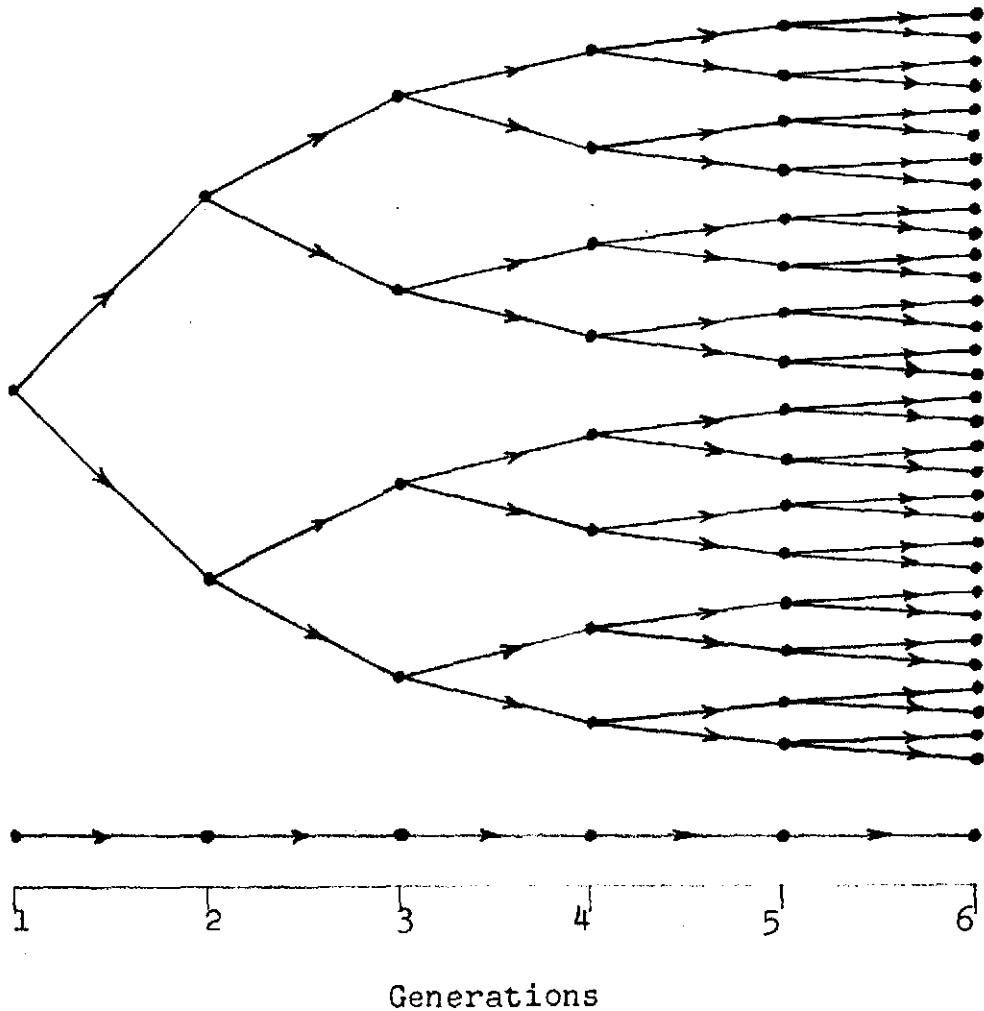


Fig. 1

Having defined the neutron multiplication factor, we can now add the following statements: -

- (1) The chain reaction is just sustained, or the reactor is just critical or the reactor power remains unchanged if  $k = 1$ .

It should be noted that the reactor can be critical at any power, whether it be 1 watt or 500 Megawatts, as long as that power remains unchanged.

- (2) The neutron population flux or neutron density increases and the power increases if  $k$  is greater than 1. The reactor is then said to be SUPERCRITICAL.

So, to increase reactor power, we have to reduce neutron losses or increase U-235 fissions to make  $k$  greater than unity, and, thus cause neutron multiplication.

- (3) The neutron population, neutron flux, neutron density and the reactor power all decrease if  $k$  is less than 1. The reactor is then said to be SUBCRITICAL, ie, below critical.

Thus, to decrease reactor power, we have to increase neutron losses to make  $k$  less than unity.

The state of CRITICALITY in a reactor is the state achieved when  $k$  becomes equal to unity and the reactor becomes critical.

### Reactivity

A reactor is critical when  $k = 1$ . The factor that would determine how subcritical or supercritical a reactor may be, is the amount by which  $k$  differs from 1. For this reason, the quantity  $(k-1)$  may be more important than  $k$  itself, especially in reactor regulation considerations. We might say that changes in  $k$  are as important, if not more important, than  $k$  itself.

Now  $k = 1$  while the reactor is just critical, but  $k$ , in a reactor, usually has a maximum value greater than unity, to permit reactor regulation and to allow for absorption of neutrons by poisons which accumulate during reactor operation. Thus  $k$  may not be 1 all the time. Changes in the value of  $k$  are referred to as REACTIVITY changes, and are normally measured in a unit called the milli- $k$  or  $mk$ , which represents one thousandth of  $k$ .

The reactivity of a reactor can therefore be used to measure how far the reactor is from being just critical.

We can therefore say that: -

- (1) the reactor is just critical when  $k = 1$
- (2) the reactor is supercritical when  $k$  is greater than one or when there is excess positive reactivity, ie, if  $k = 1.003$ , the reactor is 3  $mk$  above critical.
- (3) the reactor is subcritical when  $k$  is less than one or when the reactivity is negative, ie, if  $k = .997$  the reactor is -3  $mk$  or 3  $mk$  below critical.

For some calculations it is convenient to use a value of reactivity which is expressed as follows:

$$\text{Reactivity} = \frac{k - 1}{k}$$

For most changes in a reactor where  $k$  is very close to 1, it can be seen that there is very little difference between this value and  $k - 1$ .

It should be understood, however, that within Ontario Hydro, when a reactivity value is given, it is expressed as a change in  $k$  ( $\delta k$ ) or  $k-1$  unless it is specifically identified otherwise.

### Neutron Lifetime

The average time between successive neutron generations will decide how fast multiplication of neutrons take place. For any particular value of  $k$  or reactivity, the smaller the time between one neutron generation and the next, the faster the neutrons multiply and the faster the power increases. This time between successive neutron generations is called the NEUTRON LIFETIME.

The neutron lifetime will include the time it takes for the nucleus to fission after it has captured a neutron, the time it takes for the fast neutron, which is produced during fission, to slow down and the time taken by the thermalized neutron to be captured.

For prompt neutrons, the neutron lifetime is one-thousandth of a second (0.001 sec). The lifetime of a delayed neutron will depend on the half-life of the nucleus producing it. The average lifetime of the delayed neutrons is about 0.09 sec and this is much higher than that of prompt neutrons.

### ASSIGNMENT

1. Define the neutron multiplication factor,  $k$ .
2. In terms of  $k$ , when is the reactor
  - (a) just critical?
  - (b) supercritical?
  - (c) subcritical?
3.
  - (a) Define or explain the term "Reactivity".
  - (b) In what units will reactivity be measured?
  - (c) When  $k = 1.005$ , calculate the reactivity.

4. In terms of reactivity, when is the reactor (a) just critical?  
(b) supercritical?  
(c) subcritical?
5. (a) What is meant by "Neutron Lifetime"?
- (b) Why is there a difference between the lifetimes of the prompt and delayed neutrons and in round numbers, what are the average values of the two lifetimes?

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