Nuclear Theory - Course 127

THE CHAIN REACTION

We have already seen that there are two requirements for the useful production of power from a reactor:

1. the rate of fissioning must be high, and
2. it must be maintained continuously.

The first requirement can be met by having sufficient U-235 nuclei available in the reactor; this implies a sufficiently large quantity of fuel. The second requirement can be met if the neutrons produced at fission are made to cause further fissions, and in this way maintain a chain reaction. In this lesson, we shall consider some of the conditions which are necessary for such a chain reaction to be maintained.

Chain Reactions in Natural Uranium

To repeat the fission process over and over again, a continuous supply of neutrons must be available and the only source of new neutrons is the fission process itself. The fission process could be repeated indefinitely if some, or all, of these neutrons, produced at fission, could be used to produce further fissions. The fission process would then become self-sustaining and a chain reaction results. The minimum condition, under which a chain reaction can be maintained, is that one neutron, produced at fission, be available to cause further fission. Since 2½ new neutrons are produced during each fission, it would seem relatively easy to be able to use one of these neutrons to cause a further fission.

However, in practice it is found that if one neutron from each fission is to be available to cause another fission, a careful choice of reactor material and design is required. This is perhaps most easily illustrated by considering the feasibility of maintaining a chain reaction in natural uranium.

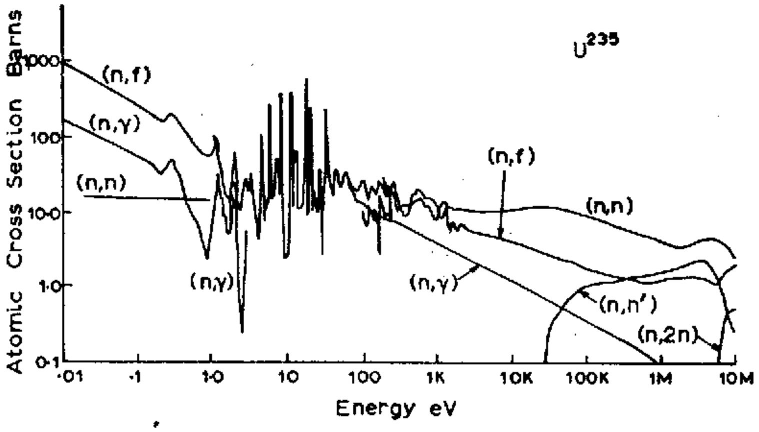

Fig.1 Macroscopic Cross Sections of U-235

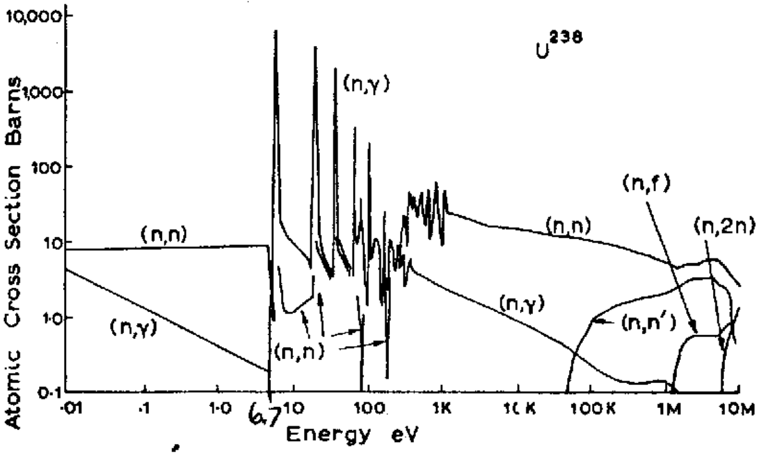

Fig.2 Macroscopic Cross Sections of U-238

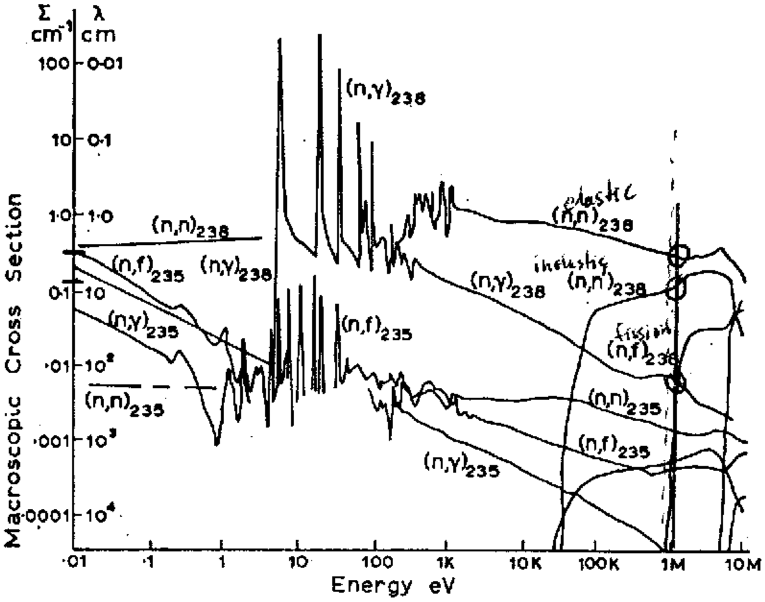

Fig.3 Macroscopic Cross Sections of Natural Uranium

n, n' inelastic scatter
n, γ rad. capt
n, n elastic scatter

Figures 1 and 2 show the microscopic cross sections of the various possible interactions with the two isotopes. In considering what happens in a block of natural uranium, these cross sections have to be weighted according to the relative abundances of the two isotopes, and this can be done by considering the macroscopic cross sections as shown in fig.3.

Below the U-238 fission threshold (~1 MeV) a neutron can only produce fission in U-235, but this is relatively unlikely as can be seen from fig.3. Above 1 MeV, fission in U-238 is possible, although not as likely as a scattering collision.

From the cross sections shown in fig.3 it is possible to obtain an indication of the fate of, say, 100 neutrons which are born in the fission process inside a large block of natural uranium. From the prompt neutrons energy spectrum (fig.4, repeated here from lesson 127.10-3), you can see that about 70 of the neutrons are born with energies above the U-238 fission threshold, and about 30 have energies below the threshold. We shall consider these 30 neutrons first.

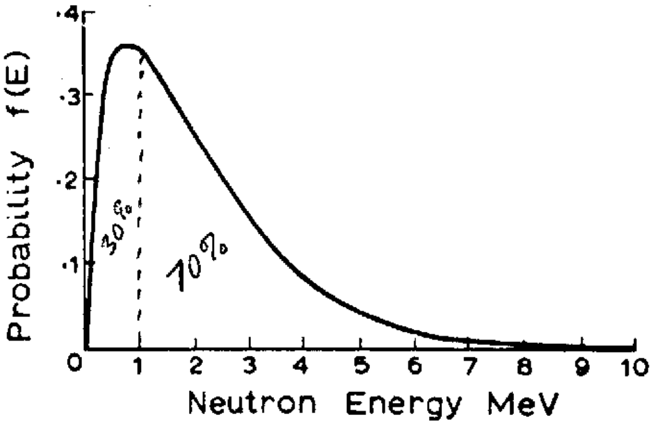

Fig.4 Prompt Neutron Energy Spectrum

From fig.3 you can see that most of the neutrons are elastically scattered, ie, $(n,n)_{238}$. Such collisions change the direction of the neutron and reduce its energy, although the reduction is small because of the large difference in mass between the uranium nucleus and the neutron. But you already know all this anyway. Inelastic scattering collisions, ie, $(n,n')_{238}$, are still possible at neutron energies down to about 100 keV, and in these collisions the neutron loses a considerable amount of energy since the uranium nucleus is left in an excited state. Therefore, after many collisions the neutrons will all be in an energy region below 100 keV and thereafter lose energy very slowly in scattering collisions. Eventually they will make a collision with a uranium atom to be absorbed, ie, $(n,\gamma)_{238}$. Since in this region the probability of capture by U-238 exceeds the probability of fission with U-235 by a factor of about 40, rather less than 1 neutron out of the original 30 produces fission - the rest are all captured by U-238.

Consider now the other 70 neutrons which are born above the fission threshold. They will undergo the various possible reactions in direct proportion to the corresponding cross sections. Therefore, from fig. 3 you can see that in their first collision about 38 neutrons are elastically scattered, in which case the neutron energy still remains above the threshold. About 27 neutrons are inelastically scattered, and for the purpose of this illustration we can assume that the loss in neutron energy is sufficient to remove them to an energy range below the fission threshold. About 4 neutrons out of the 70 undergo fission in U-238, and the remaining neutron to be accounted for is assumed to be captured in U-238, although there is also a small probability that it interacts with U-235.

The 38 neutrons which are elastically scattered make a second collision in the uranium, and the various reactions are again possible. About 2 neutrons produce fission, about 15 are inelastically scattered to an energy region below the threshold, and about 21 neutrons are elastically scattered to remain above the threshold and make further collisions.

If this sort of argument is continued to the bitter end, you will see that out of the 100 fission neutrons about 8 produce fission in U-238, and about 2 produce fission in U-235 at energies below about 0.1 MeV. Out of the original 100 neutrons, all but 10 suffer non-fission capture in U-238. If we assume that $\nu = 2.5$, 25 second generation neutrons will be produced. The *multiplication factor**) is therefore k = 0.25. This is well below the value of 1.0 which is necessary to maintain a chain reaction.

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*) The multiplication factor, k, can be defined by

$$k = \frac{\text{the number of neutrons in one generation}}{\text{the number of neutrons in the previous generation}}$$

Systems Which Will Produce Chain Reactions

Although the above reasoning undoubtedly indicates that it is not possible to produce a chain reaction in natural uranium, it does suggest ways of obtaining a chain reaction. One way is to increase the relative amount of U-235 to U-238, that is to *enrich* the fuel in the U-235 so that the chance of fission in U-235 at energies below 1 MeV ~~is~~ now increased. If the enrichment is such that out of the 88 (12) neutrons which enter this energy region at least 32 produce fission in U-235, a chain reaction can be maintained. In this case, out of the original 100 neutrons 8 produce fission in U-238, 32 in U-235 and most of the remainder are captured by U-238. These 40 fissions give rise to 40 x 2.5 100 second generation neutrons and the multiplication factor is then unity.

Fast Reactions

A reactor in which most of the fissions take place at energies above 100 keV is called a *fast reactor*. Fast reactors are described in some detail in the Reactor Boiler & Auxiliaries Course 133, and so only the barest outlines will be given here. Fast reactors require relatively highly enriched uranium or plutonium fuel (at least 20%) to increase the U-235 fast fissions. (It can be shown that about 7% enrichment of U-235 in uranium metal is the lowest possible concentration for criticality.) Furthermore, since the fission cross sections are low at high energies, a large concentration of the fuel is required to give a critical assembly - as a general rule, about half the core volume would be fuel, the rest being taken up by heat transfer and structural materials. Since enriched fuels are expensive, a high fuel concentration requires a high power density in order to bring the cost of the power down to competitive levels. Power densities in the region of a kilowatt/cm^3 are necessary, and in order to remove the heat a very efficient means of heat transfer is required, usually a liquid metal.

The production of enriched fuel is difficult and is an expensive process since it cannot be done by chemical means. Plants have been built in the U.S.A., England, France, U.S.S.R. and China using the gaseous diffusion process in which UF_6 diffuses through a porous barrier. The lighter isotope diffuses slightly faster than the heavier, and after many stages the two isotopes can be separated. The three enrichment plants in the U.S.A. between them require almost 5000 MW of electrical power.

Thermal Reactors

Because of the difficulty and expense involved in the production of enriched fuel, reactors using natural uranium have been built by adding material of low atomic mass to the uranium. The purpose of this is to cause the neutrons to lose energy by elastic scattering collisions. Provided that the material of light mass does not have appreciable absorption, the neutrons will be slowed to an energy in the region of 0.025 eV where they are in thermal equilibrium with the atoms of the material. At this energy the probability of producing fission in U-235 is high (580 b compared with 1.2 b at 1 MeV), whereas that of (n,γ) capture in U-238 is not (2.7 b). Even allowing for the low U-235 content of natural uranium, fission is now more likely than capture.

Materials of low atomic mass which do not absorb neutrons to a significant extent are called *moderators*. Reactors which contain large masses of moderating material and in which most of the neutrons reach thermal equilibrium with the moderating atoms are called *thermal reactors*.

ASSIGNMENT

1. It was said in this lesson that uranium with a minimum concentration of about 7% U-235 is necessary before a chain reaction can be maintained. Support this statement with an argument similar to that used in the text.

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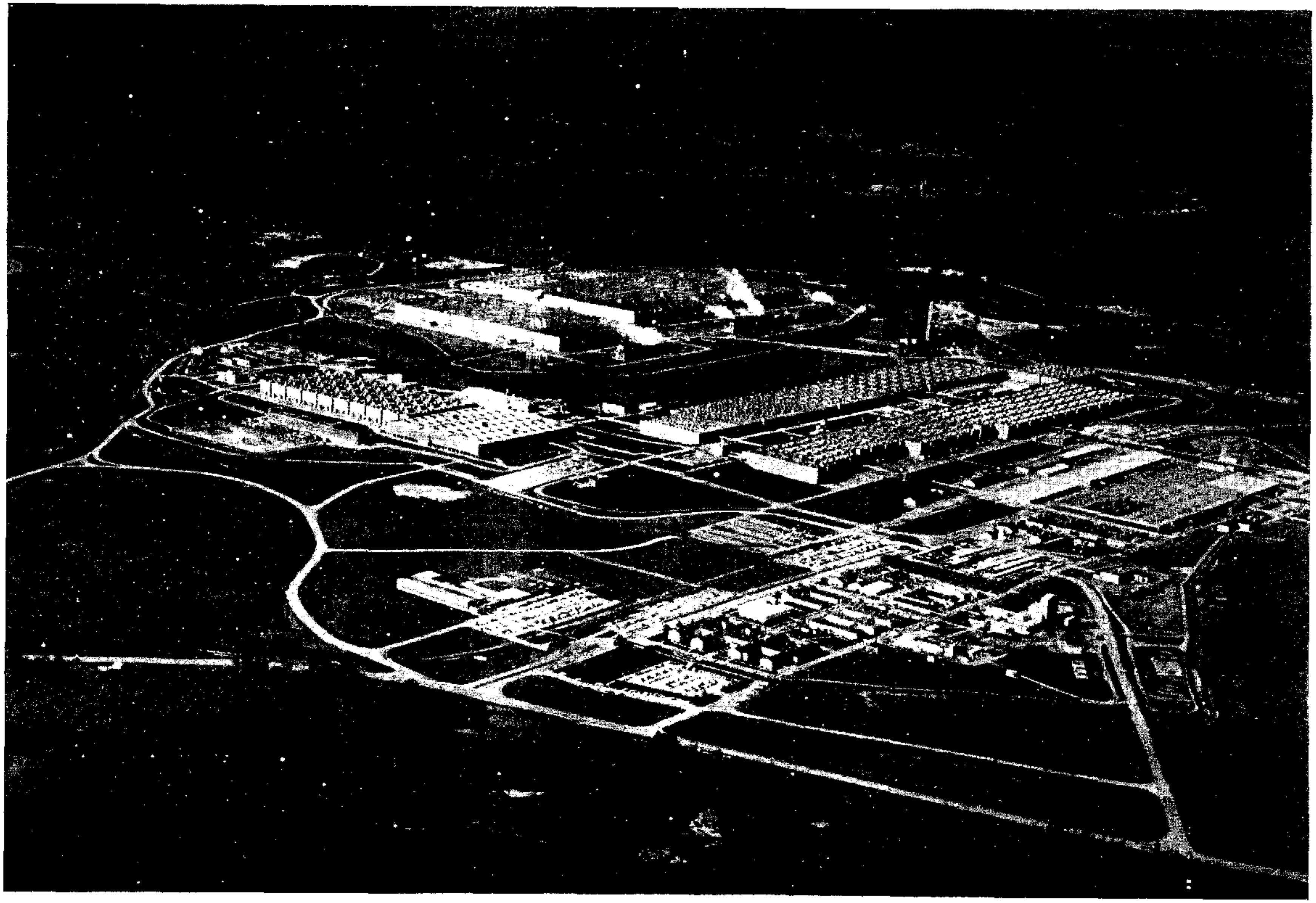

Fig. 5 Gaseous Diffusion Plant at Oak Ridge, Tennessee
(Courtesy of USAEC)