

Mathematics - Course 221

APPENDIX 1: REVIEW EXERCISES

Review Exercise #1: Reliability

1. A system of 12 dousing valves, tested monthly, has developed 10 failures of individual valves in 8 years' operation. Calculate the unreliability of an individual valve.

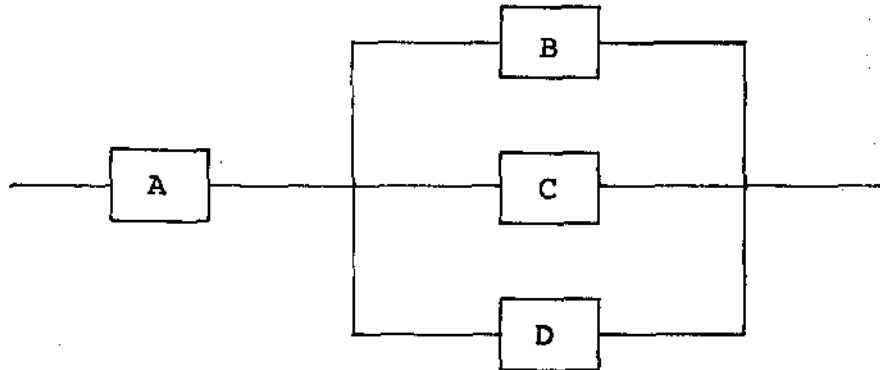
2. Calculate the annual risk of a nuclear incident at a reactor, which, during 9 years' operation, developed the following faults:
 - (a) 3 unsafe failures of the regulating system.
 - (b) 50 complete failures of the protective system, failures of which are detected and corrected at the beginning of each shift.

3. At a certain nuclear generating station, three independent divisions of equipment protect against nuclear accidents:
 - (i) process equipment with a failure frequency of 0.3 per annum,
 - (ii) protective equipment with unreliability of 2×10^{-3} , and
 - (iii) containment equipment with unreliability of 5×10^{-3} .Calculate the annual risk (frequency) of
 - (a) an incident consisting of process failure combined with simultaneous failure of either protective or containment systems.
 - (b) simultaneous failure of all three systems.

4. Monthly testing of 6 safety switches has revealed 8 failures of individual switches during 15 years' operation.
 - (a) Calculate the unreliability of a switch.
 - (b) How, without altering the equipment, could the unreliability in (a) be decreased by a factor of about 30?

4. (c) How often should the switches be tested if the permitted unreliability of a switch is 10^{-2} ?

5.



In the above system, a system failure consists of a failure of either component A, or a failure of at least two of B, C, D.

Calculate the unreliability of the system, given component reliabilities,

$$Q_A = 0.05, \text{ and}$$

$$Q_B = Q_C = Q_D = 0.1.$$

6. A pump designed for continuous operation has failed 6 times in 5 years' operation, with total down time of 124 hours. Calculate the unavailability of
- (a) the pump
 - (b) a system of three such pumps in a 3 x 50% parallel arrangement.

Review Exercise #2 (Lessons 221.20-1, 421.40-2, 321.10-3)

1. Find the equation of the following lines in the xy -plane:
 - (a) passing through $(2,4)$ and $(3,1)$
 - (b) passing through $(-4,5)$ with slope $-\frac{4}{3}$
 - (c) with y -intercept -5 and slope $\frac{2}{5}$
 - (d) passing through $(4,-1)$ and parallel to $2x - 5y + 8 = 0$
 - (e) passing through $(6,-2)$ and perpendicular to $5x + 3y - 2 = 0$
 - (f) passing through $(-5,-3)$ with angle of inclination 135°

2. Given $f(x) = x^2 - 2x + 5$ evaluate
 - (a) $f(0)$
 - (b) $f(-5)$
 - (c) $f(x + a)$

3. Plot $y = x^2 - 2x + 5$ and label the roots on the graph.

4. Find the slope and y -intercept of the following lines:
 - (a) $6x - 5y + 8 = 0$
 - (b) $\frac{2}{15}x + \frac{4}{3}y - 1 = 0$

5. For each line in Question 4 state the change in
 - (a) x corresponding to an increase of y by 2
 - (b) y corresponding to an increase of x by 5
 - (c) y corresponding to a decrease in x of $\frac{1}{2}$

Review Exercise #3 (Lessons 221.20-2, 3 and 321.10-4)

1. If $s(t) = t^2 + 2t$
- (a) plot $s(t)$ vs t
 - (b) find the average velocity over the first 2 seconds
($t = 0$ to $t = 2$)
 - (c) find the formula for the instantaneous velocity, $v(t)$,
at time t , using

$$v(t) = \lim_{\Delta t \rightarrow 0} \frac{s(t + \Delta t) - s(t)}{\Delta t}$$

- (d) find $v(2)$. (Compare with (b))
2. Find $v(t)$ and $a(t)$, using differentiation formulas if
- (a) $s(t) = -2\sqrt{t}$ (find $v(4)$)
 - (b) $s(t) = t^3 - 4t$ (find $v(5)$)

3. Differentiate with respect to x :

- (a) $5x^3 - 2x + 13$
- (c) $-x^2 + \frac{2}{x^2}$
- (b) $4\sqrt[3]{x^2}$
- (d) $\frac{x^2 - x}{\sqrt{x}}$

4. Find equation of tangent and normal to curve $y = x^2 - 2x + 3$
at (a) $x = -1$ (b) $x = 1$

5. The decay constant λ for a radionuclide is 0.010s^{-1}
- (a) Find the activity of a 10 curie source after 2.0 minutes.
 - (b) There are 3.7×10^{13} radioactive nuclei to start with.
How many remain after 2 minutes?

6. Given that a radioactive source has decay constant
 $\lambda = 1.2 \times 10^{-5} \text{ s}^{-1}$, find the
- (a) activity of a 10 Ci source after 20 hours
 - (b) half-life of the source
 - (c) time for a 10 Ci source to decay to 1 Ci
 - (d) number of radioactive nuclei in a 10 Ci source

Review Exercise #4: Practice in Solving Quadratics

1. At what points is 2 the slope of the curve

$$y = \frac{x^3}{3} + x^2 - 13x + 10?$$

2. At what instants does the velocity equal zero if

$$s(t) = 2t^3 + \frac{11t^2}{2} - 10t - 5?$$

3. At what x values are the slopes of the two curves equal?

(a) $y_1 = x^3$; $y_2 = x^2 + x$

(b) $y_1 = 3x^3 + \frac{19}{2}x^2$; $y_2 = 12 + 15x - \frac{x^3}{3}$

4. At what instants are the velocities equal for the following displacement functions?

$$s_1(t) = t^3 + \frac{5}{2}t^2 - 3t ; \quad s_2(t) = \frac{t^3}{3} + 3t^2 + 4$$

Review Exercise #5: (To end of lesson 221.20-4)

1. Using data tables, find $\ln x$ for $x = .001, .01, .1, .5, 1, 2, 4, 6, 8, 10$. Use your table of values to plot a graph of $\ln x$ versus x .
2. Find (a) $\ln e^{2.5}$ (b) $\log \frac{1}{10,000}$
(c) $\log 10^{4/t}$ (d) $\ln e^{-\lambda t}$
3. If $A(t) = A_0 e^{-\lambda t}$ prove that half-life $t_{1/2} = \frac{0.693}{\lambda}$
4. The decay constant for a radionuclide is $3.8 \times 10^{-5} \text{ s}^{-1}$. Find the
 - (a) half-life
 - (b) number of half-lives to decay from 8 Ci to 1 Ci.
 - (c) time for activity to die down to 1% original
 - (d) time for activity to die from 2 Ci to 100 μCi
 - (e) number of radioactive nuclei in a 2 Ci source
(1 Ci = 3.7×10^{10} dps)
5. Given $\Delta k = .008$, $L = 0.25$, $P_0 = 10$ watts, $P(t) = P_0 e^{\frac{\Delta k}{L} t}$
 - (a) find $P(t)$, $P'(t)$ at $t = 20$ minutes
 - (b) If rated power were 1000 MW, find the time to reach rated power.
6. Given $P(t) = P_0 e^{\frac{\Delta k}{L} t}$ show that rate log power is directly proportional to $\frac{\Delta k}{L}$. Explain the importance of a rate log meter to reactor operation.
7. Differentiate with respect to variable x or t as applicable:
 - (a) $\frac{6}{\sqrt{x}}$ (b) $2x^3 - 11x^2 + 14$ (c) $-\sqrt{x} + \frac{2}{x}$
 - (d) e^{2x^2} (e) $100e^{-4t}$ (f) $e^{2/t}$

Review Exercise #6: (To end of lesson 221.20-4)

1. Differentiate: (a) x^{15} (b) $\frac{-2}{\sqrt{x}}$ (c) $\frac{x^2-x}{\sqrt[3]{x}}$
(d) $e^{\sqrt{x}}$ (e) e^{-1/x^2}
2. Find $v(t)$ and $a(t)$ given
(a) $s(t) = 50t - 9.8t^2$ (b) $s(t) = e^t + t$
(c) $s(t) = 2t^3 - 14t^2 + 5t - 8$
3. Find equation of tangent and normal to $y = x^2 - 6x - 16$ for
(a) $x = -1$ (b) $x = 3$
4. Find equation of the line
(a) through $(-2,5)$ and $(6,-1)$
(b) through $(4,1)$ and having y -intercept of -3 .
5. Find slope and y -intercept of $4x + 5y - 13 = 0$.
6. Find the roots of $f(x) = x^2 - 6x - 16$.
7. Given $\Delta k = .005$, $L = .12$ s, $P_0 = 50$ W, find, using $P(t) = P_0 e^{\frac{\Delta k}{L}t}$,
(a) reactor power after 2 minutes
(b) time for reactor to gain from 1% to 100% power.
8. Find (a) $\ln e^{-t^2}$ (b) $\log 10^{\sqrt[4]{t}}$
9. Given $t_{1/2} = 10$ minutes, find
(a) decay constant (b) time for source to decay from 0.5 Ci to 10 μ Ci
10. Find the activity of a source consisting of 2.0×10^{15} radioactive atoms if the decay constant is $6.5 \times 10^{-4} \text{ s}^{-1}$ in
(a) dps (b) curies

Review Exercise #7: Integration Problems (Lesson 221.30-1)

- Find the area under the following curves between the indicated limits:
 - $y = 2x + 1$, $x = 0$ to $x = 5$
 - $y = \sqrt{x}$, $x = 4$ to $x = 9$
 - $y = \sqrt[3]{x^2} + 3x + 2$, $x = 8$ to $x = 27$
- Find (i) $v(t)$ (ii) $s(t)$ given the following:
 - $a(t) = -2$, $v(0) = 6$, $s(0) = 0$
 - $a(t) = 2\sqrt{t}$, $v(0) = 0$, $s(0) = 100$
 - $a(t) = -t + 3$, $v(0) = v_0$, $s(0) = 0$
- Given $\frac{dy}{dx} = 4x + 5$ find y
 - Given $\frac{ds}{dt} = t^{3/2}$ find $s(t)$
 - Given $\frac{dv}{dt} = 6t$ find $v(t)$
- Find $v(t)$ and $s(t)$ given
 - $a(t) = -9.8 \text{ m/s}^2$ $v(0) = v_0$, $s(0) = 0$
 - $a(t) = 0 \text{ m/s}^2$ $v(0) = v_0$, $s(0) = 0$
 - $a(t) = \sqrt{t} \text{ m/s}^2$ $v(0) = v_0$, $s(0) = 10 \text{ m}$
- Integrate:

(a) $x^2 - 2$	(b) $2t^3 - 4t$
(c) \sqrt{x}	(d) t^{-5}
(e) $\frac{2}{x^2}$	(f) $\frac{5}{\sqrt{x}} + 14$
- Find the displacement function $s(t)$ if the velocity function is
 - $v(t) = 2t - 3$
 - $v(t) = 3\sqrt{t} + 4$
- Find $v(t)$ and $s(t)$, given
 - $a(t) = -5 \text{ m/s}^2$, $v(0) = 10 \text{ m/s}$, $s(0) = 0$
 - $a(t) = 2t^2$, $v(0) = 0$, $s(0) = 0$

Review Exercise #8: (To end of lesson 221.30-1)

1. Given $f(x) = x^3 + x^2 - 17x + 15$. Graph $y = f(x)$, and label the roots of $f(x) = 0$.
2. Find the roots of $f'(x) = 0$, given $f(x)$ as in Question #1. What is the significance of these roots to the curve $y = f(x)$?
3. Find the equation of the tangent and normal to $y = f(x)$ of Question #1 at $x = 1$.
4. Evaluate

(a) $e^{\ln 0.1}$	(b) $10^{\log t^2}$	(c) $\ln e^{-2/t}$	
(d) $\log 10^y$	(e) $-\ln e^{-0.4}$	(f) $10^{\log 100}$	(g) $e^{\ln \lambda t}$
5. A source consisting of 8.6×10^{13} radioactive atoms is decaying at the rate of 7.5×10^9 dps. Find
 - (a) the decay constant
 - (b) the half-life
 - (c) the time required for the activity to die to 1 μCi .
6. If the half-life of a radionuclide is 8.4 minutes, find
 - (a) the decay constant
 - (b) the time for source activity to decrease by a factor of 1000.
7. Differentiate:

(a) $x^7 - 6x^3 + \sqrt[3]{x}$	(b) $\frac{x^3 - 1}{\sqrt{x}}$
(c) $\sqrt[3]{x^2}$	(d) $x^{2/5} + \frac{a}{x}$
(e) e^{-2/t^2}	(f) e^{x^2-4}
(g) $\sqrt{x}(x^3 - \frac{1}{x})$	

8. If reactor power builds up from 100 W to 1000 MW in 5.0 minutes and the mean time between neutron generations is .1 seconds, find the reactivity.
9. If $N(t) = N_0 e^{-\lambda t}$, prove that $\frac{dN}{dt} = -\lambda N$.
10. Find the equation of the line
- (a) parallel to $5y - 2x + 3 = 0$ and passing through the origin
 - (b) perpendicular to $5y - 2x + 3 = 0$, and having the same y-intercept
 - (c) having an angle of inclination of 45° and the same x-intercept as $5y - 2x + 3 = 0$.
11. (a) Given $f'(x) = -2x + 0.4$, find $f(x)$ if $f(0) = -7$.
- (b) Given acceleration $a(t) = \frac{2}{\sqrt{t}}$, $v(0) = 1$, $s(0) = 4$, find $v(t)$, $s(t)$.
- (c) The R/C of $g(x)$ with respect to x is $-\frac{a}{x^2} - 10$. Find $g(x)$.
- (d) y increases 3 times as fast as x . If $y = -5$ when $x = 0$, find y as a function of x .
12. Find the area under the curve
- (a) $y = x^3$ from $x = 1.5$ to $x = 5$
 - (b) $y = e^x$ from $x = 0$ to $x = 3$
 - (c) $y = \sqrt[3]{x}$ from $x = 1$ to $x = 8$

Review Exercise #9: (To end of Lesson 221.30-2)

1. Plot $y = 1.5^x$, $-5 \leq x \leq 10$.
2. Plot A vs t where $A(t) = A_0 e^{-\lambda t}$, $A_0 = 100$ Ci, and $\lambda = 0.01 \text{ s}^{-1}$
 - (a) using semi-log graph paper for $10 \leq t \leq 1000$ s.
 - (b) using linear graph paper for $0 \leq t \leq 1000$ s.

(Note the advantages/disadvantages of logarithmic graph paper.)
3. The force F to extend a spring varies directly as the extension x in meters.

ie, $F = kx$, where k is called the spring constant

 - (a) Prove that the work done in stretching the spring x meters equals $\frac{1}{2}kx^2$.
 - (b) How much work is done in stretching a spring by 0.25 m if its spring constant is 1.2×10^4 N/m?
4. The force of gravity, F_g , on a satellite of mass M_s varies inversely as the square of its distance x from the earth's centre, ie,

$$F_g = \frac{GM_e M_s}{x^2}$$

where G is the universal gravitation constant, and M_e is the earth's mass.

 - (a) Prove that the work done by a rocket to lift a satellite d meters above the earth's surface (neglecting friction) is

$$W = \frac{GM_e M_s d}{R_e (R_e + d)}$$

where R_e is the earth's radius.
 - (b) How much energy must the rocket provide to free the satellite from earth's gravity altogether?
5. Translate the following rate-of-change statements to differential equations:
 - (a) The torque T on a wheel equals the product of the wheel's moment of inertia I times the time rate of change of the wheel's angular velocity, ω .

(b) The voltage V across an inductor equals the product of the inductance L times the rate of change of the current i with respect to time.

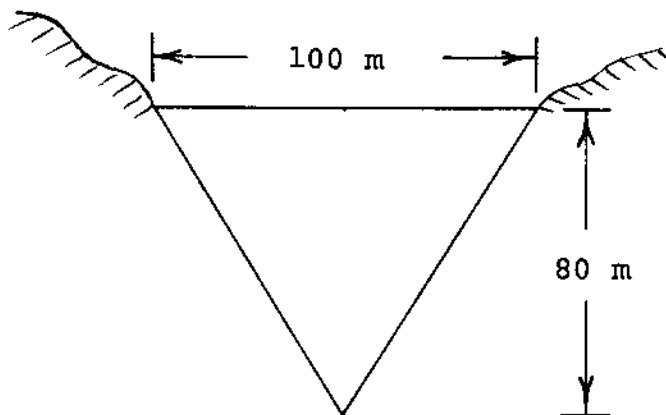
6. For a poison-injection shut down of a reactor, gadolinium (Gd) is injected into the moderator at a concentration of 20 mg Gd/kg D_2O . This Gd must be removed before the reactor can be restarted. Moderator cleanup is achieved by cycling the moderator through ion exchange columns, and the concentration $C(t)$ of Gd remaining in the moderator after t hours is given by the expression.

$$C(t) = 20 e^{-0.35t} \text{ mg Gd/kg } D_2O$$

Find:

- (a) the time required to reduce the concentration to 0.8 mg Gd/kg D_2O , at which the reactor can be restarted.
- (b) an expression for the rate at which Gd is removed, as a function of time.
- (c) the total reduction in Gd concentration during the first 10 hours.
- (d) the average rate of Gd concentration during the first 10 hours
- (e) the instantaneous rate of Gd removal at half-time ($t = 5$ h). Why is this rate different from that of (d)?
- (f) the average concentration during the first 10 hours.
7. For the V-shaped hydraulic dam illustrated below, assuming water level coincides with top of dam, find:
- (a) the total force exerted by the water against the face of the dam
- (b) the average pressure of the water against the dam face
- (c) the center of pressure exerted by the water.

7.



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