5 Activity and Half-Life

The activity of a radioactive material is the number of nuclei that decay per unit time, often expressed as "disintegrations per second". In the SI system, this unit is called the becquerel (Bq). The becquerel is defined as one radioactive disintegration (decay) per second. Another unit, widely used, is the curie (Ci). One curie is now defined as 3.7×10^{10} Bq. Originally, it was the activity of 1 g of radium-226. The discovery of radium won Marie Curie the first of two Nobel prizes.

5.1 The Law of Radioactive Decay

A pure radioactive substance decays at a fixed fractional rate. That is, in each second a constant fraction of the total amount present decays. Consequently, the actual number of atoms decaying per unit time is proportional to the amount of the substance.

Consider a particular sample of a radionuclide. The continual decay decreases the quantity of the sample and so the activity decreases. This process continues until the radioactive material is gone. Figure 5.1 plots the quantity of the sample Q against time T.

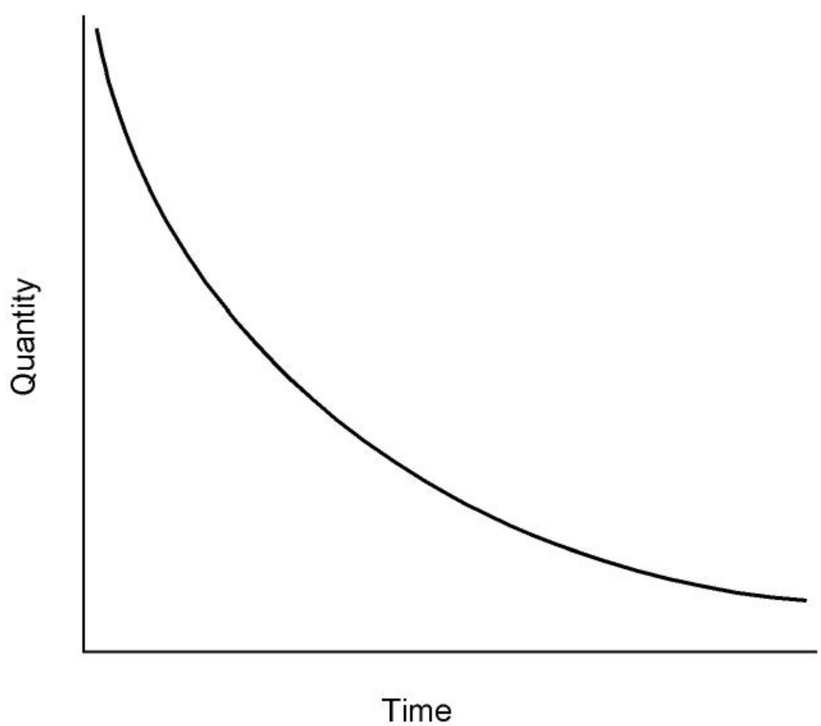

Figure 5.1
Quantity v Time for a Radionuclide

We have already said that activity is proportional to the quantity of the radioactive substance so we can also plot the activity against time (Figure 5.2).

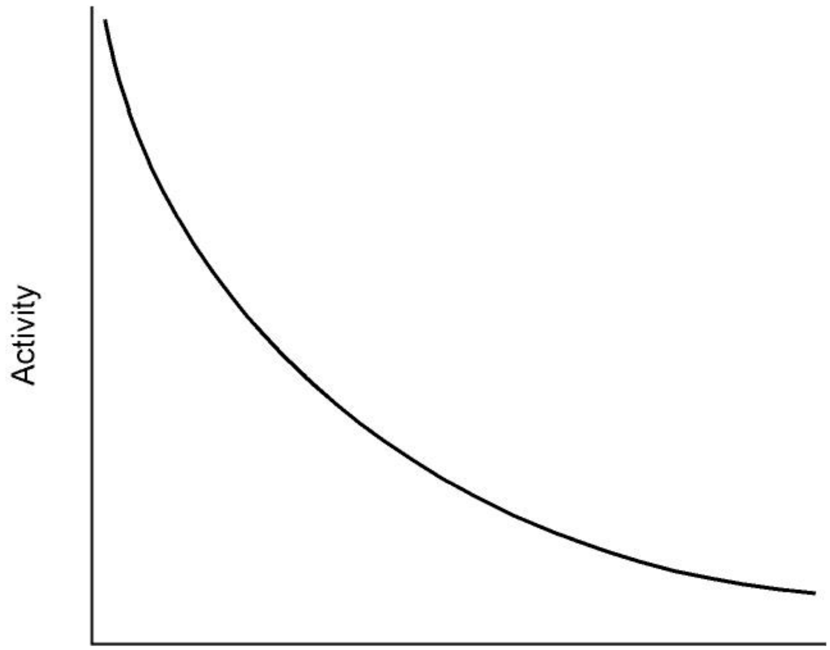

Figure 5.2
Activity v Time for a Radionuclide

These two graphs (Figures 5.1 and 5.2) are mathematically identical and only differ in having different vertical scales. In practice, we prefer to use the second form because activity is the quantity usually measured and activity is usually what interests us most. By contrast we seldom can measure, and often don't care about the actual quantity of the radioactive substance. For example, the activity of the moderator quoted in curies per kilogram gives a clear indication of the radiation hazard, but tells us nothing directly about the amount of tritium in the moderator.

5.2 Half-life

If we plot graphs of activity vs. time for different radioactive materials, we find they have different rates of decay (Figure 5.3). To distinguish between the different rates, we use half-life (Figure 5.4). The half-life ($T_{1/2}$) is the time interval for the activity of a specimen to fall to half of its original value.

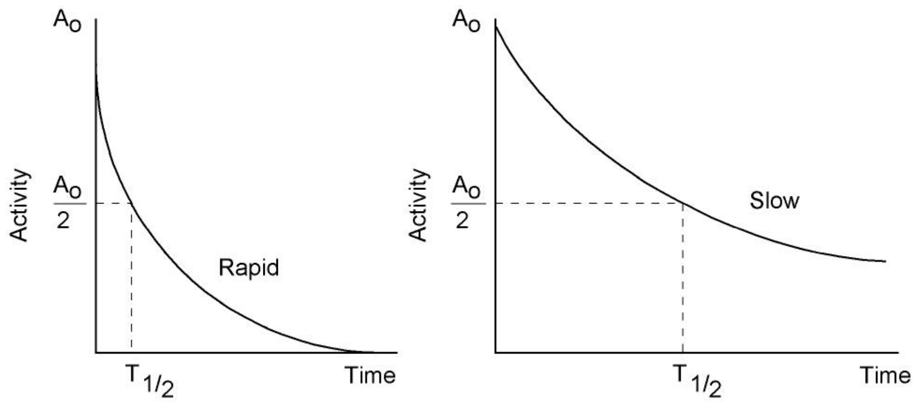

Figure 5.3 A and B -
Activity Plots of Two Different Radioactive Materials

The time interval between activity A_0 and activity ½ A_0 is, by definition, one half-life. For an exponential decay curve (which these are), it does not matter where we start with A_0. For any starting point on the curve, the time for the activity to decrease to ½ the starting value is the same (Figure 5.4).

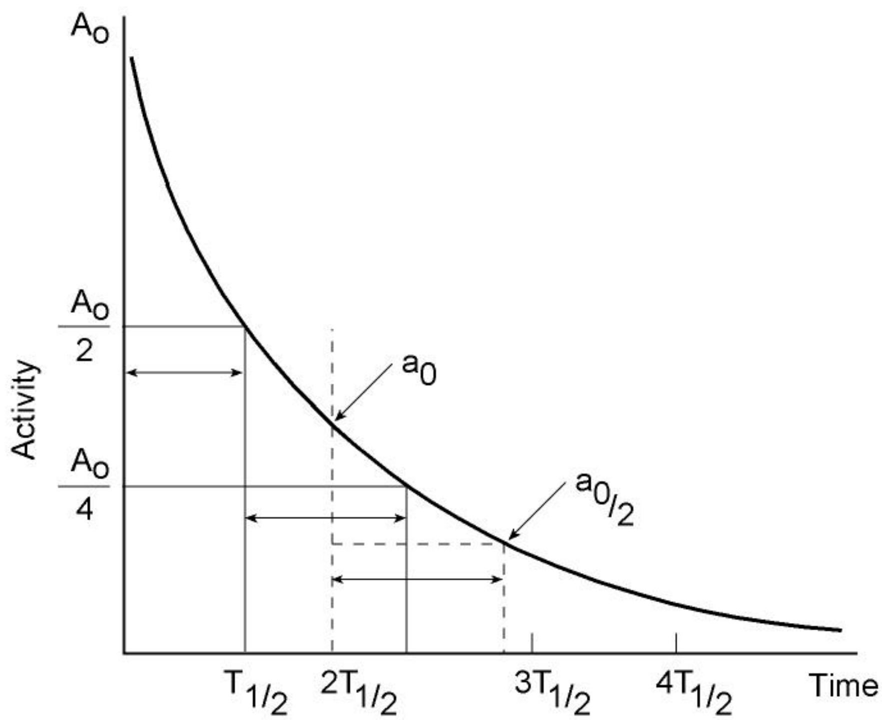

Figure 5.4 –
Activity and Half-life

The time from A_0 to ½ A_0 is the same as from ½ A_0 to ¼ A_0. It also takes the same time from a_0 to ½ a_0. This leads to the formula At=A_0 (1/2) n where n is the number of half-lives, i.e., n = t/$T_{½}$ where t is the elapsed time. In this relationship, we usually take n to be an integer but it need not be.

Another form of the equation At=A_0 $(1/2)^n$ is A_0/A_t = 2 n

Before reviewing the following examples, try the end-of-chapter-exercises. Many people find these calculations easier to do than to read about.

Examples

1. Suppose a radioactive substance has an activity of 6144 Bq. How many half-lives will it take for the activity to fall to 6 Bq?

   $$\frac{A_0}{A_t} = 2^n$$

   Substituting the values, $\frac{6144}{6} = 2^n$, or, $1024 = 2^n$

   $\therefore \quad n = 10$

   Answer:

   It takes 10 half-lives for the activity to fall from 6144 to 6 Bq.

2. What will the activity be 6 half-lives later for a radioactive substance which has an activity now of 192 Bq?

   $$A^t = A_0(1/2)^n$$

   $$A_t = 192(½)^6 \qquad i.e. \ (192 \ x \ ½ \ x \ ½ \times ½ \times ½ \times ½ \times ½)$$

   $$\text{or} = \frac{192}{2^6} = \frac{192}{64}, \qquad \text{so} \qquad A_t = 3\text{ Bq}$$

   Answer:

   After 6 half-lives an original activity of 192 Bq falls to 3 Bq.

3. If the half-life in the first example is 25 minutes, what is the time t?

$t = n\,T_{½}$

$= 10$ half-lives x 25 minutes per half-life

$= 250$ m or 4h 10 m

5.3 Range of Half-lives

Half-lives vary from very short (fractions of a second) to very long (billions of years). From an operational point of view, these values are important in comparison to reactor life, operational times, outage times, fuel life, etc.

For example, fresh CANDU fuel contains natural uranium. The half-life of U-238 is 4.5 billion years and for U-235 is 700 million years. Although both of these are decaying by α emission, we never notice any change in their activity over the lifetime of the reactor. Fresh fuel will be the same no matter how long we keep it. By contrast, the half-life of N-16 (produced by activation in the reactor core) is only 7s and the activity changes faster than most of us can calculate.

Irradiated fuel contains isotopes of uranium and neptunium that decay to make fissile plutonium. The beta decays of U-239 and Np-239 have half-lives of 23m and 2.3d respectively. They convert into fissile Pu-239 on a quite short time scale. The Pu-239 decays by α decay with a half-life of 25 thousand years so its quantity does not change due to α decay in the 1 to 2 years the fuel is in the reactor.

Activity (rate of decay) depends on half-life, so it too has a wide range of values. The becquerel is a very small unit of activity and is usually seen with a metric prefix. For example: kilo becquerel (k Bq = 10^3 Bq), mega becquerel (M Bq = 10^6 Bq), giga becquerel (G Bq = 10^9 Bq) or even, sometimes, tera becquerel (T Bq = 10^{12} Bq). The curie, on the other hand, is a large unit. It is customary to see it as milli curie (m Ci = 10^{-3} Ci), micro curie (μ Ci = 10^{-6} Ci), pico curie (p Ci = 10^{-12} Ci), or, occasionally, nano curie (n Ci = 10^{-9} Ci). As an example, contamination with about 8×10^{-9} g of radioactive I-131 (about 3.7×10^{13} atoms of I-131) results in an activity of about 1 m Ci = 37 M Bq.

5.4 Summary of Key Ideas

- Activity is the rate at which a radioactive material is decaying.

- Activity is measured in d.p.s., Bq or Ci.
- Nuclei decay exponentially and each has a characteristic half-life.
- Half-lives vary over a tremendous range, from tiny fractions of a second to billions of years.

5.5 Assignment

1. Give the relationship between disintegrations per second and the becquerel.

2. Name a widely used base unit for activity other than the becquerel.

3. Fe-59 has a half-life of 45 days. If a sample has an activity of 1000 decays per second, what is its activity after one year?

4. The activity of a radioactive specimen is 2×10^7 Bq. After 20 days, the activity is 2×10^4 Bq. What is the half-life of this specimen? (Calculate to the nearest whole number of half-lives.)