Fluid Mechanics - Course 223.0

BASIC QUANTITIES AND UNITS

Volume

The basic unit in SI is the cubic meter - m^3. If smaller units are required a litre - L which is $\frac{1}{1000}$ m^3 may be used.

Density

Defined as mass per unit volume - kg/m^3 - symbol ρ (rho). Density describes the quantity of matter or material contained within a particular volume.

Typical figures for density:

Name	Density (kg/m^3)
Mercury	13546
Gasoline	750
Water	1000
D_2O	1100

Consider this problem - A tank of volume 2.6 m^3 has to be filled with D_2O. How many kilograms of D_2O are needed? Check your answer at the end of the course.

Similarly a pipeline holds 37924 kg of water - What is the volume of the line?

Relative Density

Defined as a ratio of density of the material compared with that of water - no units - symbol 'd'.

$$d = \frac{\text{Density of Substance}}{\text{Density of Water}}$$

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The density of water is conveniently taken to be 1000 kg/m^3 - thus the previous typical values for density now become as follows for relative density:

Name	Relative Density
Mercury	13.546
Gasoline	0.750
Water	1.0
D_2O	1.1

We can see that with the density of water being 1000 kg/m^3 the expression for relative density may be easily modified to give the density of the substance:

'd' x 1000 = density of substance

Consider the following example - An oil tank contains 5000 m^3 of oil with relative density of 0.85. Determine how many kg of oil are contained in the tank? Check at the end of the course.

Consider a pipeline of 30 cms internal diameter and 120 m long. The line holds kerosene of relative density 0.78. How many kg of oil are in the line? Check at the end of the course.

Velocity of Flow

Usually describes the rate of flow of fluid in a system, eg, fluid which flows past two points which are 5 m apart in one second, has a velocity of 5 m/s. Velocity of flow is a good initial indicator as to whether high system losses may be expected due to high velocities.

Volumetric Flowrate

This is the volume which flows past a particular point, every second - units m^3/s - symbol Qv. We can determine the volumetric flowrate, knowing the internal diameter of the line and the velocity of flow, eg, oil flows along a line, which is 15 cm internal diameter, at a velocity of 6.5 m/s - determine the volumetric flowrate.

Volumetric flowrate = Sect. Area of line x velocity

$$\text{Sectional area} = \frac{\pi D^2}{4}\ m^2$$

$$= \frac{\pi \times 0.15^2}{4}$$

$$= \underline{0.0177\ m^2}$$

Thus $Q_v = 0.0177 \times 6.5$

$= 0.1149\ m^3/s$

Try this example - water flows along a line at 0.22 m^3/s. The line is 25 cms I.D. - calculate the velocity of flow. Check at the end of the course.

Volumetric flowrate is normally used when density changes are very small. Any system that has no significant temperature changes may use volumetric flowrates for liquid flow. If there are significant changes in temperature then the volume per unit mass does not remain constant and we must choose another parameter to indicate flow. The mass of fluid in a system, ignoring leakages and additions, will remain constant but the volume that the mass occupies varies with temperature. This happens in the feedwater system. The temperature at the condensate extraction pump outlet is around 38°C whilst at the final high pressure heater outlet, the temperature is around 175°C.

You may also have noticed that the flowrate is measured in kilograms per second. This is the MASS FLOWRATE and indicates the quantity of material passing a point per unit time. The mass flowrate has the symbol Q_m and the units arem, of course, are kg/s.

A useful relationship connects the volumetric and mass flowrates:

$Q_m = \rho \times Q_v$

$kg/s = kg/m^3 \times m^3/s$

Oil of relative density 0.915 flows along a line having an I.D. of 35 cms. The velocity of flow is 5.6 m/s - calculate

(a) the mass flowrate

(b) the volumetric flowrate.

Check your answers at the end of the course.

Heavy water flows along a line which is 30 cms I.D. at 400 kg/s. Determine the velocity of flow along the line.

Pressure

May be simply defined as force per unit area. In SI units the force is in newtons and the area is in square meters. Thus pressure

$$P = \frac{N}{m^2}$$

and these units are called PASCALS (Pa). Standard atmospheric pressure is 101 300 Pa(a). Note the pascal is a small unit and will frequently be encountered in multiples of 10^3 (kPa) and 10^6 (kPa).

Before we get involved with some problems we'll define some basic pressures.

(a) Absolute Pressure (Pa(a))

Pressure measured from a datum of a perfect vacuum - thus all absolute pressures are ALWAYS positive.

(b) Atmospheric Pressure

Pressure due to the weight of the earth's atmosphere acting on the earth's surface - measured in Pa(a).

(c) Gauge Pressure (Pa(g))

Pressure measured from a datum of atmospheric pressure - thus may be either positive or negative.

(d) Vacuum (-Pa(g))

This is a pressure which is measured using atmospheric pressure as the datum. It is always below atmospheric pressure.

There is no real need for all these commonly quoted forms of pressure. Absolute pressure would cover all these considerations. However, we are faced with these different forms on a day to day basis and should know how they are related. A simple diagram will relate these pressures relatively quickly.

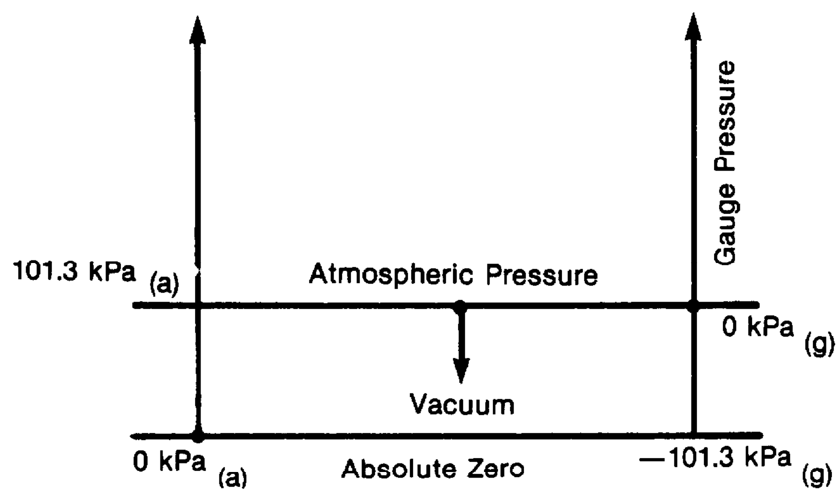


Figure 1

Suppose we have a pressure which is 30 kPa below atmospheric pressure. Using the diagram we can describe this single pressure in three ways:

(a) 30 kPa Vacuum

(b) -30 kPa (g)

(c) 71.3 kPa(a)

Convert these pressures to gauge pressure:

(a) 84 kPa(a)

(b) 22 kPa Vacuum

(c) Atmospheric pressure

(d) 143 kPa(a)

Convert these pressures to absolute pressure:

(a) -46 kPa(g)

(b) Atmospheric Pressure

(c) 247 kPa(g)

(d) 89 kPa Vacuum

Check the answers at the end of the course.

Variation of Pressure with Height

Consider a column of liquid which is h meters high and which has a sectional area A as shown. The pressure at the depth h is due to the weight of the column of liquid.

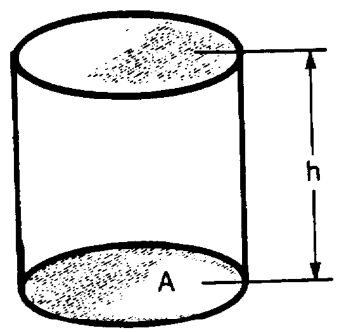


Figure 2

The volume of the liquid is A x h m^3. The mass of the liquid = density x volume

= ρ x A x h kg.

weight = Mass x 'g'.

Thus the weight of the liquid column = ρ x g x A x h N

Pressure is force per unit area. The weight of the column of liquid is acting on an area of 'A' square meters.

Thus the pressure = $\dfrac{\rho \times g \times A \times h}{A}$ Pa

= $\rho \times g \times h$ Pa

If we know the density of the fluid and the depth, the pressure may be easily determined.

A tank which is 15 m high contains oil of relative density 0.85 to a depth of 9.4 m. Determine the pressure exerted on the tank floor due to the weight of the liquid in the tank. See end of course for the answers.

An oil tank has 60 cms of water on the bottom. The kerosene tank level is 6.4 m. The relative density of kerosene is 0.78 - calculate the pressure on the tank floor due to the liquid. Check your answers at the end of the course.

Consider a pipeline full of water at a pressure of 80 kPa(g). If we connect a tube to the pipe the pressure in the pipe will push water up the tube. The figure illustrates the problem.

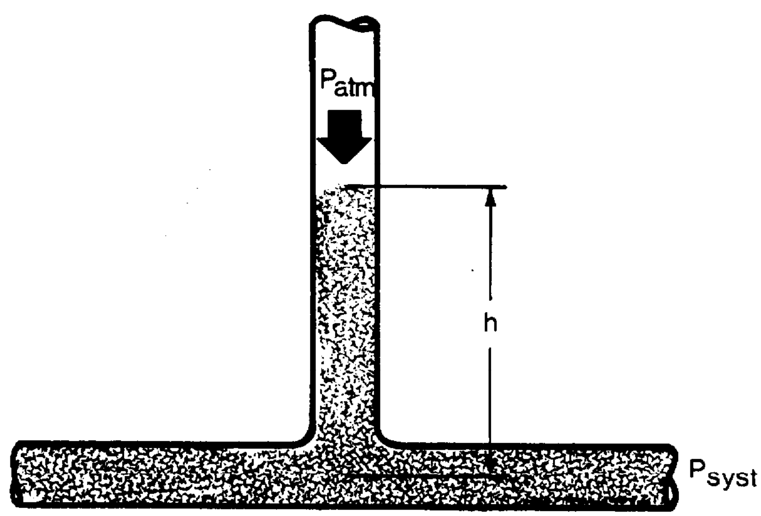


Figure 3

The tube, which is connected to the pipe is open to atmosphere. The pressure in the pipe, P_{SYSTEM} is supporting a column of liquid height 'h'. If we write a simple equation for the pressure in the system we get P_{SYST} = Pressure due to the column of water + P_{ATM}. The pressure due to the column is ρgh.

So $P_{SYST} = \rho gh + P_{ATM}$

Rearranged $\rho gh = P_{SYST} - P_{ATM} = P_{SYST}(g)$

Thus $\rho \times g \times h = 80 \times 10^3$ Pa

$$\therefore \quad h = \frac{80\ 000}{1000 \times 9.81} = 8.15 \text{ m}$$

This is the height of the water column that would exist with P_{SYST} = 80 kPa(g)

We could also measure the pressure by exactly the same technique using a manometer.

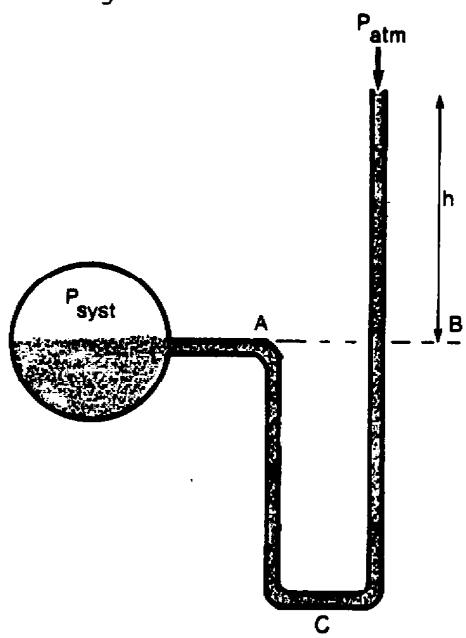


Figure 4

Consider the diagram showing a manometer connected into the line.

If we start from point C where the pressure is common to both legs A and B we can demonstrate that the pressure at point A is equal to the pressure at point B. This has to be, because if you agree that PC is common to both legs and the heights of A and B above C are equal, then the pressure drops from P_C to P_A and P_B are equal and consequently $P_A = P_B = P_{SYST}$. Consider the pressure at B. If we recall the previous example we can clearly see that:

$P_b = \rho \times g \times h$ Pa(g) $= P_{SYST}$.

The suction pressure to a pump is measured using a manometer. The level of water, in the leg of the manometer open to atmosphere, is 2.3 m below the level in the system leg. Sketch the arrangement and calculate the suction pressure.

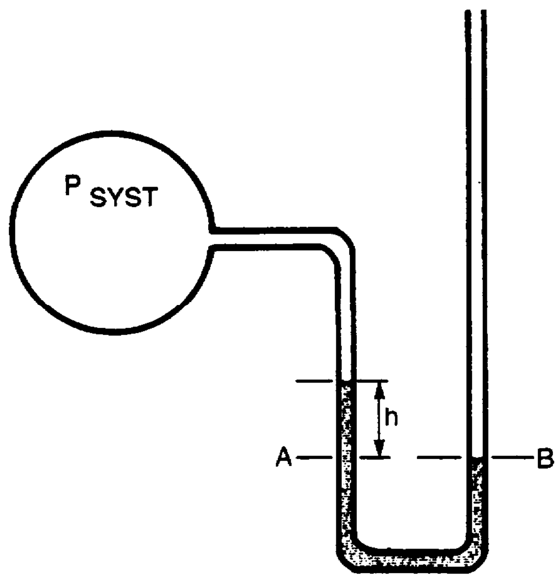


Figure 5

If the system pressure is above atmospheric then the liquid level in the atmospheric leg will be above that in the system leg. Conversely, if the system pressure is sub-atmospheric the level in the atmospheric leg will be below that of the system leg.

Again we can see that

$P_A = P_B$

$PA = P_{SYST} + \rho \times g \times h$ (we can ignore the height of the vapour or air column because the very low density makes this mathematically insignificant.

$P_B = P_{ATM}$

Thus

$$P_{SYST} = P_{ATM} - \rho \times g \times h = 101\ 300 - 1000 \times 9.81 \times 2.4$$

$$= 101\ 300 - 23\ 544$$

$$= 77\ 756 \text{ Pa(a)}$$

A condenser is operating at a pressure of 6 kPa(a). The condenser pressure is to be checked with a manometer using mercury (d = 13.546). Sketch the arrangement and show the liquid levels. Calculate the level difference you would expect to find. Check your answers at the end of the course.

A tank is to be hydraulically pressure tested to 300 kPa(g). The pressure is to be measured using a mercury filled manometer, relative density of mercury is 13.546. Sketch the arrangment of the manometer showing liquid levels and calculate the difference in levels you would expect to find. Check your answers at the end of the course.

In the diagram of the reactor calandria, the heavy water is maintained at a level of 8 m by the pressure difference between the helium gas pressure in the dump tank and the helium gas pressure in the upper space of the calandria. Calculate the pressure difference

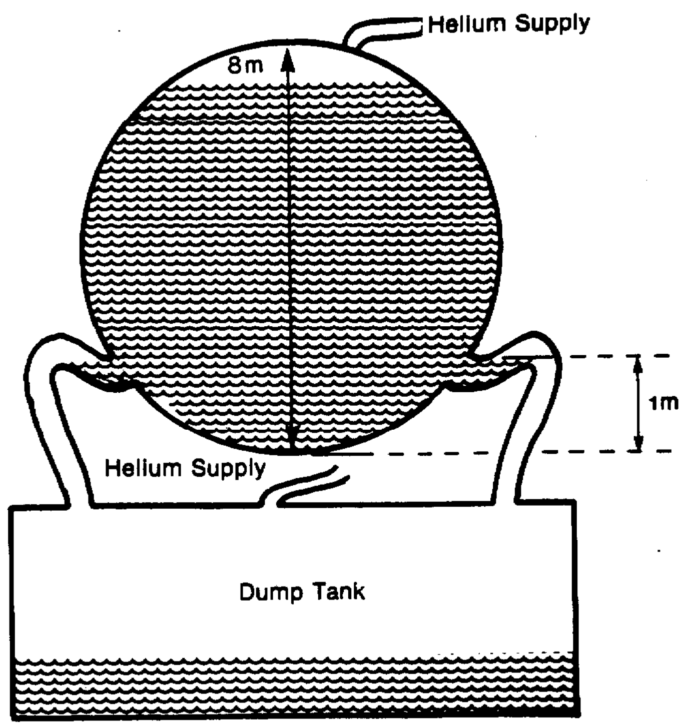


Figure 6

between the dump tank and the cover gas. Relative density of heavy water is 1.1 and the density of light water is taken as 1000 kg/m^3.

N.B. Whilst the reasons for the pressure in the cover gas system are not directly part of this course, they are often included with a question on this topic.

Two Phase Lever Systems

The same principle may be used for the remote indication of the water level in a two-phase pressurized system such as a conventional steam drum.

Consider the diagram below.

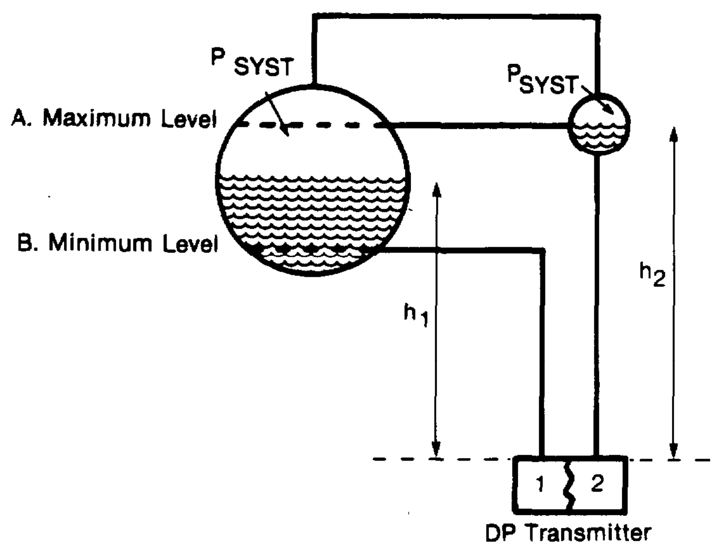


Figure 7

The steam which is fed to side 2 of the P transmitter will tend to condense in the pipework to the transmitter and form a head of liquid, which, if not held constant, will seriously affect the differential pressure readings. It is necessary to ensure that any liquid condensing in the piping to side 2 is a constant quantity. Before we analyze the level measurement a description of the arrangement is required. The small tank on the righthand side of the diagram situated at the maximum level provides the constant height of liquid which is used for the reference. The liquid is supplied by the condensing of the vapour from the steam drum via the connecting pipe which is usually sloped towards the steam drum.

Any excess liquid drains back into the steam drum ensuring a constant level in the reference leg, the uppermost line is to ensure equal pressures in the small tank and the steam drum. This line is often ommitted as the condensate return pipe will also provide pressure equalization.

At the minimum level position on the drum, another line is connected. The liquid line from the reference tank runs down to a cell which measures differential pressure. The liquid line from the minimum level position on the drum is connected to the other side of the DP cell.

The reference pressure at side 2 in the DP cell has two components, the system pressure and the pressure due to the height of the column of liquid.

Thus $P_2 = P_{SYST} + \rho \times g \times h_2$.

Similarly $P_1 = P_{SYST} = \rho \times g \times h_1$.

The DP cell has P_2 on one side and P_1 on the other - the difference in pressure between $P_2 - P_1$ providing the signal to the transmitter.

$$P = P_2 - P_1.$$

$$= P_{SYST} + \rho \times g \times h_2 - P_{SYST} - \rho \times g \times h_1$$

$$= \rho \times g \; (h_2 - h_1)$$

Thus the signal from the DP cell depends upon the difference between the maximum level and the actual level.

In practice the DP transmitter is usually biased so that it produces a maximum reading when the P is zero. This does not affect the rationale of the hydraulic circuit.

Let's examine the pressures acting on the DP cell when the tank is at the maximum level. At maximum level

$$P_1 = \rho \times g \times h_1 + P_{SYST} \text{ and } P_2 = \rho \times g \times h_2 + P_{SYST}$$

$$\text{but } h_1 = h_2$$

Thus $\Delta P = P_2 - P_1 = 0$, and the transmitter is biased to produce a maximum reading corresponding with the maximum level.

Similarly if the minimum level exists P_1 will now be $\rho g h_1 + P_{SYST}$ and the ΔP will be a maximum of $\rho g (h_2 - h_1)$, although, again the transmitter will be biased to produce a minimum reading corresponding to the minimum level.

Explain what level would be indicated

(a) if the level drops below point B.

(b) if the level rises above point A.

Check your answers at the end of the course.

The arrangement shown is typical of the AECB questions. Explain how a test reading may be obtained corresponding to the maximum level in the tank.

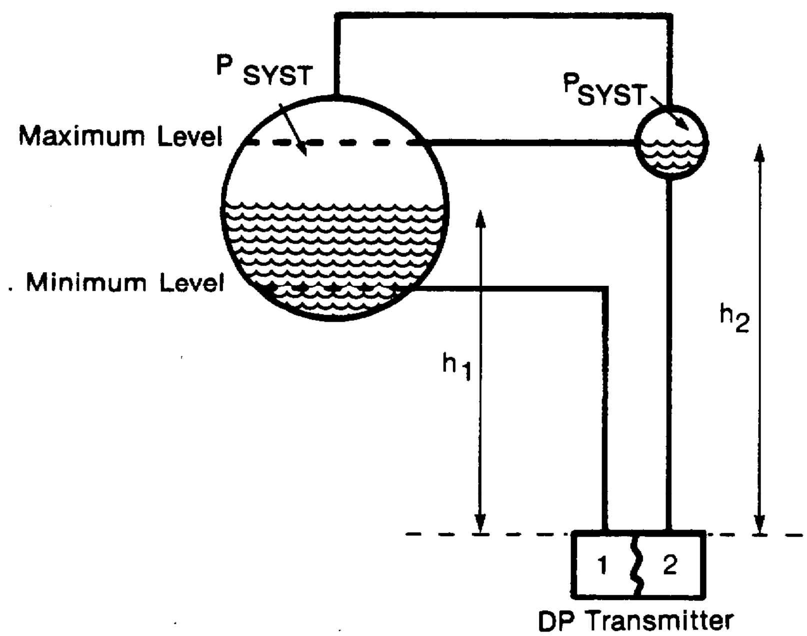


Figure 8

Check your answers at the end of the course.

Again on this question involving a two phase pressurized system another variation involves modification of the circuit to allow response tests.

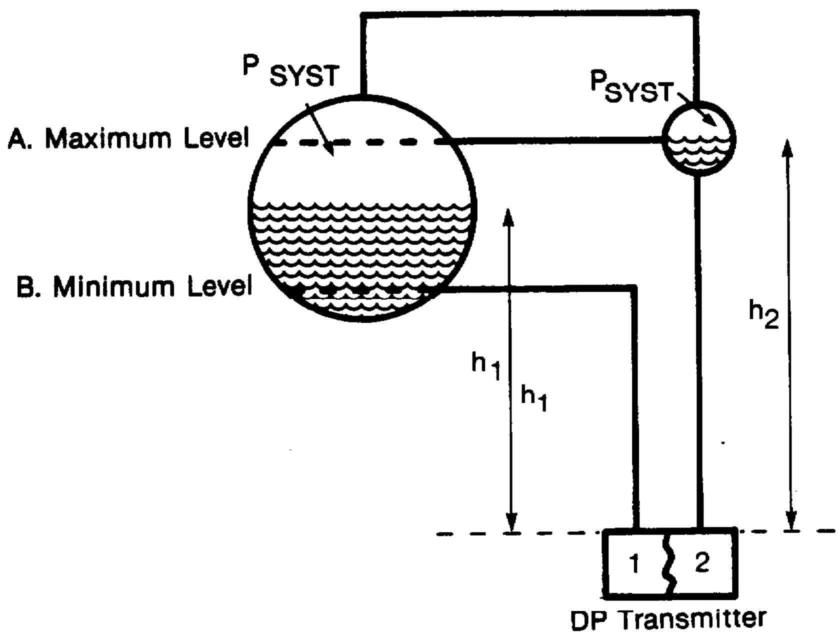


Figure 9

Draw a possible arrangement of additional lines and valves that would permit checking the proper response of the transmitter at both the maximum and minimum level. Using this arrangement, describe the procedure for performing the response test. Note: make certain that your arrangement permits restoring normal, correct, operation after the tests are performed. Check your answers at the end of the course.

We have examined how pressure changes with height, we must now examine how pressure may be used to create a force and produce work. From our basic definition of pressure being force per unit area, ie, $P = F/A$, we may re-arrange this expression so that $F = P.A$. This may be applied in the application of a hydraulic ram where hydraulic pressure 'P' is provided under a piston of area 'A' and provides a force 'F'.

A ram of 10 cms diameter is supplied with oil at a pressure of 600 kPa(g). Calculate the force produced by the ram.

$$\text{Area of the ram} = \frac{\pi D^2}{4} = \frac{\pi \times 0.1^2}{4} = 0.00785 \text{ m}^2$$

$$\text{Thus } F = P \times A = 600 \times 10^3 \times 0.00785$$

$$= \underline{\underline{4712 \text{ N}}}$$

Consider the following question:

The diagram represents a compressed air actuated hydraulic lift. A pressure of 200 kPa(g) of air in cylinder A is needed to support the load L on the piston in cylinder B.

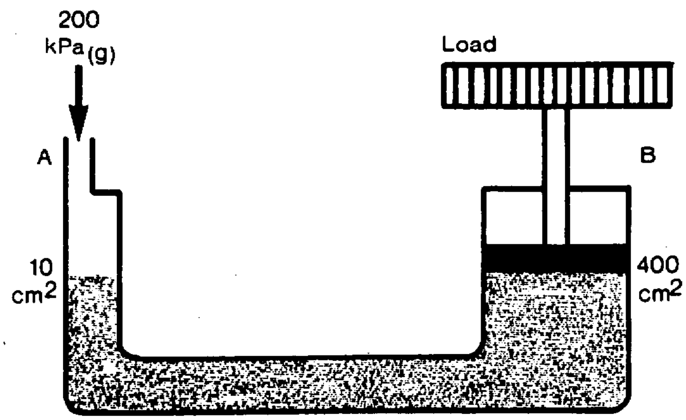


Figure 10

Determine the weight of the supported load showing your reasoning and calculations.

If this load was raised 2 m in two minutes at a constant rate, determine the power output (ignore friction).

If the load was raised 2 m how far will the liquid fall in cylinder A.

Check the end of the lesson for your answers.

In a dynamic fluid system where fluid is moving, the basic energy source is pressure energy. This may have to satisfy the following requirements:

(a) establish velocity of flow.

(b) overcome frictional effects.

(c) overcome increases in elevation.

(d) establish minimum required system pressure.

Thus if pressure is the only energy source we may describe the conditions by writing an equation for all the energies: thus

Pressure = Velocity + Friction + Elevation + Pressure.

Consider the familiar event of opening a pump discharge valve which is closed. What happens to the pressure? It decreases! Why? - because as the fluid starts to flow the velocity increases so the pressure falls. In addition as velocity increases the frictional effects increase and the pressure falls further on this account. Shut the valve, eliminate the flow and the pump discharge pressure is a maximum.

Consider the following example of an 8" SCH 40 line with a divergent section such that the line becomes 16" SCH 40. The pressure upstream from the divergent section is 360 kPa(a). Consider the pressure at the point just downstream from divergent section. The flowrate is 0.45 m^3/s. The pressure downstream is

(a) less than 360 kPa(a)

(b) more than 360 kPa(a)

(c) 360 kPa(a)

Check the end of the course for the answers.

A pipeline supplies a flowrate of 0.4 m^3/s. The line varies in section. Explain what you would expect to find at the small and large sections when considering velocity and pressure. Check the end of the course for the answers.

In any pumping system some of the energy input to the pump appears as heat due to the churning and frictional effect of the impellor within the liquid. Leave a pump running on a closed discharge - what happens. Sure! the pump becomes hot fairly quickly. This situation is not unique to the closed discharge situation, the heating is being produced all the time the pump is running. Under normal flow conditions we do not witness such a large rise in temperature of the pump casing because the heat is carried away with the working fluid.

The amount of energy that is converted to heat directly across the pump reduces the pump efficiency.

If a pump has an efficiency of 65%. What we are stating is that of the energy input - 65% appears as <u>pressure</u> energy across the pump and 35% appears as heat energy across the pump.

A pump is driven by a motor having an output of 5 MW. The pump is 60% efficient and the motor is 85% efficient. How much heat appears across the pump?

(a) 3 MW

(b) 2 MW

(c) 5 MW

(d) 4.24 MW

(e) 2.55 MW

Check your answers at the end of the course.

Let's consider a pump running on a <u>closed loop</u>. Suppose that the suction and discharge pipework is the same diameter so that the velocity at the suction to the pump is the same as the velocity at the discharge.

Again, suppose that there is no elevation difference between the suction and discharge piPework. The discharge pressure is 4 mPa(g) and the suction pressure is 1.5 mPa(g). What happened in the system to cause the pressure to fall from 4 mPa(g) to 1.5 mPa(g)? Check your answers at the end of the course.

A primary heat transport pump is 68% efficient and is supplied with energy at the rate of 3.6×10^8 Joules/min. Describe how the pump input energy is distributed throughout the system and calculate the power values. Check your answers at the end of the course.

In most systems the flowrate is variable, whether by means of control valves, multiple pump arrangements or changing liquid levels. The flowrate is a function of the pressure difference which exists between the supply point and the discharge point. The function which relates flowrate and pressure is by no means simple. As the velocity increases the kinetic energy associated with the flowrate increases as the $(\text{velocity})^2$.

In addition the frictional effects in the hydraulic circuit also tend to vary as the $(\text{velocity})^2$ but are more complex because the friction factor, which measures the effect of the pipe roughness with flow, also changes, consequently a doubling of the system pressure will <u>not</u> double the flowrate. If we plot the pressure required for various flowrates through a specific circuit, we will have a rising characteristic, ie, as the flowrate increases the pressure required to maintain this flowrate also increases but at a disproportionate rate.

This curve is commonly termed the "system head" curve and describes the flow characteristic for various system pressures.

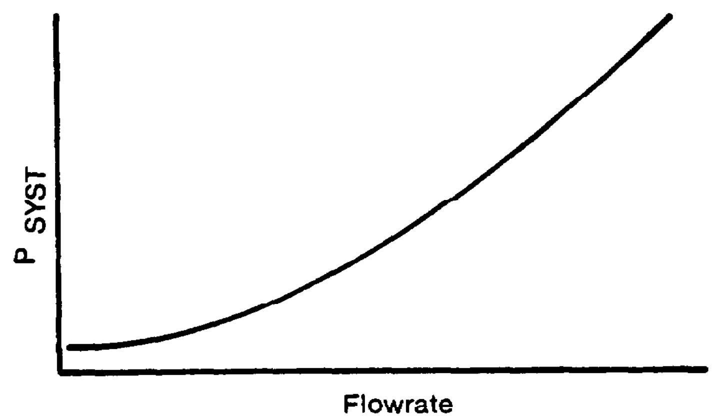


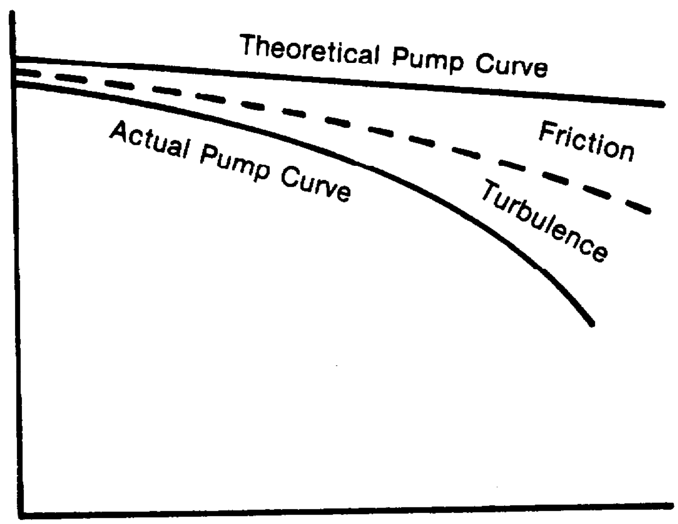
Figure 11

By exactly the same process we can produce a curve which is a result of plotting the discharge pressure from the pump which is recorded at various flowrates.

In theory, for a given pump and speed, the pump discharge pressure varies linearly with the flowrate. The usual design of a centrifugal pump has blades curved backwards at an angle of <90° which produces a linear falling characteristic. For blades which are radial at the exit, ie, the angle = 90°, the theoretical discharge pressure is independent of flowrate. For blades curved forward, at an angle >90°, this produces a characteristic where the discharge pressure rises with increasing flowrate.

As previously mentioned the usual design is to have blades angles less than 90°C which produces a linear falling characteristic.

In practice two major losses disturb the linear characteristic. Fluid friction increases as the square of the flowrate causing loss of discharge pressure. Secondly, the pump can only be designed for one flow-rate, at a given speed. This is the point of best efficiency and turbulence and shock losses are negligible. For flow rates above and below the design, turbulence reduces the available pressure. The resulting characteristic is a falling curve and not a straight line.



Pump Characteristic for a Single Pump

Figure 12

In a fixed hydraulic system where the characteristic is constant, there can only be one flowrate for a given system pressure. If we draw a diagram of the pump characteristic with the system curve superimposed, we can examine what occurs on start up.

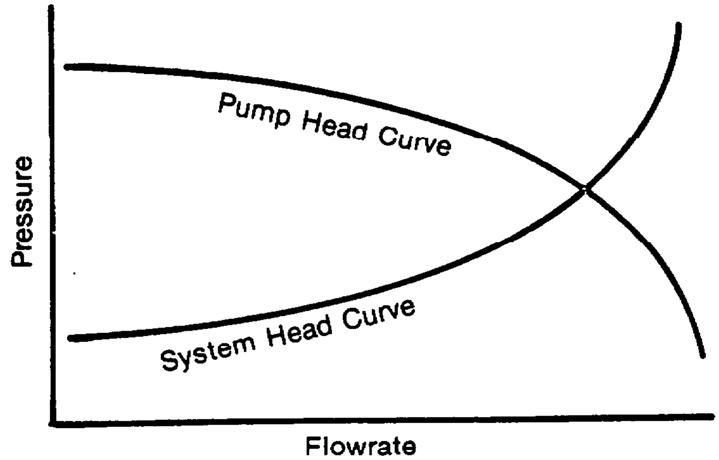


Figure 13

Consider the system with the pump running but with the discharge valve closed so that there is no flow. With no flow through the pump there is no friction loss due to flow velocity (although friction losses occur within the pump due to churning) and no pressure energy required to establish kinetic energy consequently the discharge pressure is a maximum. When we examine the system head curve, there is no flowrate so we do not need any pressure from the pump to establish flow.

By gradually opening the discharge valve the pump pressure may be applied to the system. As the flowrate through the pump and system increase two major effects on pressure may be observed. As the kinetic energy and friction through the pump increases with increasing flow the available pump discharge pressure decreases. As the flowrate through the system increases, the pressure required to maintain flow increases due to the KE and friction. Thus with the flowrate increasing, the available pressure from the pump is falling whilst the pressure required by the system is rising. At the point at which the pump head curve intersects the system head curve, the pressure produced by the pump is equal to that required by the system for that flowrate and stable flow occurs.

Explain, using the pump and system head curves, what happens to the system curve if a valve in the system is partially closed. Check your answers at the end of the course.

Let's consider the same system fed with two identical pumps in parallel, both having the same stable pump head characteristic.

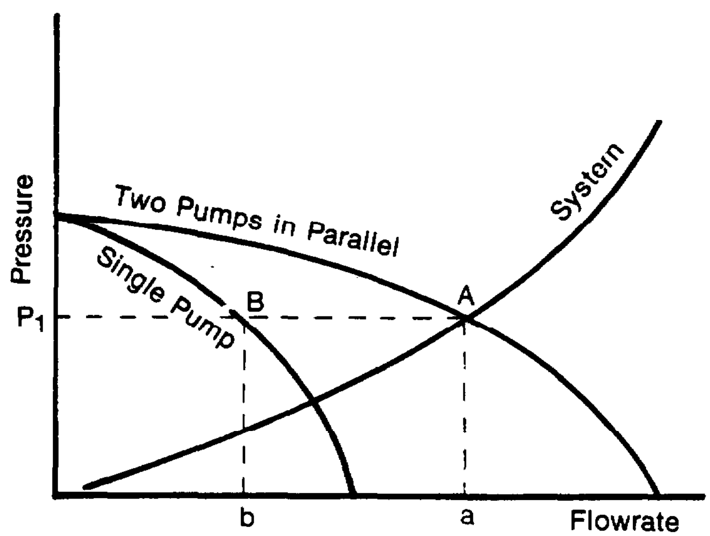


Figure 14

The first point to note is that the maximum pressure does not increase with two pumps operating in parallel. Secondly, the system flowrate is equally shared by the two pumps. If we consider the flowrate at point A this is generated by both pumps providing a discharge pressure of P_1 and an individual flowrate 'b', ie, 2 x 'b' = 'a'.

Consider the above system with three pumps in parallel and suggest how the flowrate will change. Explain your reasoning using the curves shown. Check your answers at the end of the course.

If one of the pumps, in the arrangement using two pumps in parallel, was tripped, discuss what would happen to the flowrate using the head curve diagrams fo illustrate your answer. Check your answers at the end of the course.

If your require extra assignment questions, past AECB examination papers will yield a number of similar problems, all of which may be answered using the same rationale that has been employed in this text.

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